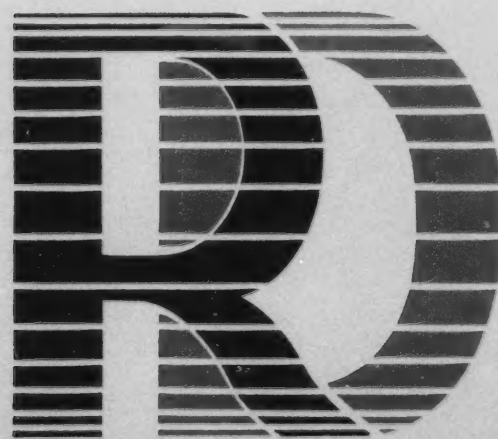
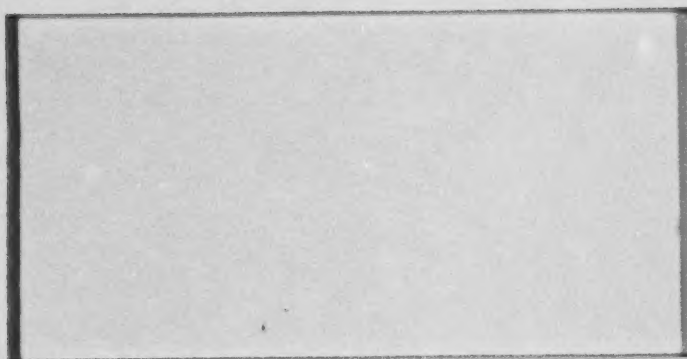


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IRRADIATED FUEL SHOCK AND  
VIBRATION ENVIRONMENT DURING  
OFF-SITE TRANSPORTATION

Report No 83-482-K

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ABSTRACT

This report describes an analytical and experimental study to define the shock and vibration environment during off-site transportation of irradiated fuel. Two modes of transportation are considered; namely road and rail. Digital computer simulation and modal testing are used to determine the normal off-site transportation shock and vibration input from the vehicle/tiedown configuration as well as the shock and vibration transmission through the module structure. The results indicate that vertical preload of the module stack is not necessary during road transportation. During rail transport, the predicted shock response spectra at the module tubes due to railcar coupling impact indicate that vertical restraint on the module stack may be required if the fuel bundle impact tests predict bundle failures for such a transient shock.

| job            | file  | date              | report no. |
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EXECUTIVE SUMMARY

IRRADIATED FUEL SHOCK AND VIBRATION  
ENVIRONMENT DURING OFF-SITE TRANSPORTATION

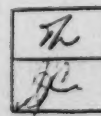
M.A. Elbestawi and D.F. Lau  
Plant Equipment Dynamics Section  
Mechanical Research Department

An analytical and experimental study has been performed to define the shock and vibration environment during off-site transportation of irradiated fuel.

A mathematical model has been developed to predict the random response of a tractor-semitrailer system carrying a nuclear fuel shipping cask to road undulations. The model allows the flexibilities associated with the cask, tiedown, and transport frame to be simulated. Digital computer simulation is used to predict the vibration input to the shipping module from the vehicle/tiedown configuration.

The shock environment occurring during coupling of a railcar carrying a nuclear fuel shipping cask was also investigated by computer simulation. Results include acceleration time and frequency profiles at selected points in the transportation system.

The shock and vibration transmission through the module was investigated with a three-dimensional experimental modal analysis of two storage and transportation modules stacked one on top of the other. Ten modes of vibration were identified in the 0 to 1000 Hz range. The natural frequency, shape and associated damping of these modes were used to define the modal data base of the module stack. The excitations predicted by analytical modelling were used along with the experimentally defined modal data base of the module stack to predict the shock and vibration transmission through the module.



| job            | file  | date              | report no. |
|----------------|-------|-------------------|------------|
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Some of the main conclusions of this study are:

1. The simulation of the vibration environment during road transportation indicates that vertical preload of the module stack is not necessary.
2. The predicted shock response spectra at the module tubes due to a coupling impact of a railcar carrying a road cask indicate that preload of the module stack may be necessary if the fuel bundle impact tests predict failure for such a transient shock.

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# NOMENCLATURE

|                          |                                                                                   |
|--------------------------|-----------------------------------------------------------------------------------|
| $T, U, D$                | Kinetic, potential, and dissipative energy terms in Lagrangian equation           |
| $M, K, C$                | Mass, stiffness, and linear (viscous) damping matrices                            |
| $E$                      | Young's modulus of Elasticity                                                     |
| $t$                      | Plate thickness                                                                   |
| $\nu$                    | Poisson's ratio                                                                   |
| $[K_e]$                  | Element stiffness matrix                                                          |
| $[M_e]$                  | Element mass matrix                                                               |
| $[C_e]$                  | Element damping matrix                                                            |
| $[T]$                    | Transformation matrix                                                             |
| $[\lambda]$              | Matrix of Direction Cosines                                                       |
| $[M_c]$                  | Condensed mass matrix                                                             |
| $[K_c]$                  | Condensed stiffness matrix                                                        |
| $[C_c]$                  | Condensed damping matrix                                                          |
| $\alpha, \beta$          | Scalar quantities used in calculating proportional damping in the tiedown members |
| $b_{if}, b_{ir}, b_{ij}$ | Vehicle dimensions                                                                |
| $c_f$                    | Forced damping matrices                                                           |
| $c_{if}, c_{ir}$         | Suspension damping rates                                                          |
| $c'_{if}, c'_{ir}$       | Tire damping rates                                                                |
| $f$                      | Frequency in Hz                                                                   |
| $\bar{f}$                | Vector of excitation forces                                                       |
| $H(f)$                   | Transfer function                                                                 |
| $h(f)$                   | Impulse response function                                                         |

|                    |                                                                          |
|--------------------|--------------------------------------------------------------------------|
| $h_{ij}$           | Vehicle dimension                                                        |
| $i$                | $\sqrt{-1}$                                                              |
| $K_f$              | Forced stiffness matrix                                                  |
| $k_{if}, k_{ir}$   | Suspension spring rates                                                  |
| $k'_{if}, k'_{ir}$ | Tire spring rates                                                        |
| $l_{jk}$           | Spacing between $j$ th and $k$ th wheel                                  |
| $n$                | Spatial frequency of road roughness in c/m                               |
| $n_0$              | $1/2\pi$ c/m                                                             |
| $P$                | Transformation matrix                                                    |
| $R_F(\tau)$        | Force correlation function matrix                                        |
| $R_W(\tau)$        | Temporal correlation function matrix                                     |
| $R_X(\tau)$        | Response correlation function matrix                                     |
| $R_W(\delta)$      | Spatial correlation function matrix                                      |
| $R_1, r_2$         | Waviness                                                                 |
| $S_F(f)$           | Force spectral density function matrix                                   |
| $S_W(f)$           | Temporal spectral density function matrix                                |
| $S_X(f)$           | Response spectral density function matrix                                |
| $S_W(n)$           | Spatial spectral density function matrix                                 |
| $s(n)$             | Spatial spectral density of road roughness                               |
| $s(n_0)$           | Roughness coefficient; the value of the road Sepctrum at frequency $n_0$ |
| $t$                | Time                                                                     |
| $\tau_{jk}$        | Time delay                                                               |
| $\bar{w}(t)$       | Vector of temporal excitation displacements                              |

|                  |                                            |
|------------------|--------------------------------------------|
| $v$              | Vehicle velocity                           |
| $\bar{w}(x)$     | Vector of spatial excitation displacements |
| $w_j(t)$         | Temporal displacement function             |
| $\{\bar{w}(x)\}$ | Stationary Gaussian random process         |
| $w_j(x)$         | Spatial displacement function              |
| $\bar{x}$        | Vector of generalized coordinates          |
| $x_j$            | Element of vector $\bar{x}$                |
| $\bar{z}$        | Vector                                     |
| $\beta_{jk}$     | Phase angles                               |
| $\delta$         | Distance lag                               |



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IRRADIATED FUEL SHOCK AND VIBRATION  
ENVIRONMENT DURING OFF-SITE TRANSPORTATION

1.0 INTRODUCTION

The analytical analysis of the shipping cask/storage and transportation modules/tiedown system response to various anticipated transportation environments is a part of the Irradiated Fuel Transportation Shock and Vibration (S&V) Program as outlined in /1/.

The objective of the overall S&V program is to ensure that the irradiated fuel bundles and module container will not fail from shock and vibration during normal transportation via road, rail or water to an interim or permanent storage site/1/.

The road and rail modes of transport are examined in this report for the off-site transportation of irradiated fuel.

The objectives of the work presented in this report are:

- 1.1 The determination of the normal off-site transportation shock and vibration input from the vehicle/tiedown configuration.
- 1.2 Determination of normal off-site shock and vibration transmission through the module.

2.0 LITERATURE SURVEY

The scope of this literature survey pertains to the response of nuclear fuel shipping containers to shock and vibration loads which will be present during normal off-site transportation. Accident conditions will not be considered. Modes of transportation considered are road and rail.

Dynamic excitations delivered to shipping containers are defined in /2/ as a mixture of vibration, occasional shock superimposed on the vibration and isolated shock which occurs in single events such as rail coupling. Vibration excitation is produced by the carrier suspension system and frame members reacting to travel over surface irregularities in roads or railtracks. For trucks the superimposed shock is produced by crossing railroad tracks, bridge approaches etc. For railcars the shock resulting from rail coupling operations is by far more severe than that which is superimposed and intermixed with the vibration (eg, those produced by crossing bridges, switches, and other track intersections).

A summary of data from the ERDA/DOD and ERDA Transportation Data Banks was compiled by Sandia Laboratories to define the vibration and shock environments encountered during truck transportation/2/.

The vibration data compiled were obtained from seven different truck and tractor-trailer configurations. The trucks were equipped with both conventional spring suspension systems and air-cushioned suspension systems. The cargo weights varied from no load to 133 000 N (15 tons). Speeds varied over different road types. The acceleration amplitudes were measured at the interface between the cargo and the cargo floor.

Table 1 shows the data compiled by Sandia representing 0 to peak acceleration amplitude levels which envelope 99% of all amplitudes measured in each frequency band.

The remaining one per cent of the data was considered to represent superimposed shock. The shock data were obtained from measurements taken when the trucks encountered pot holes, railroad crossings, and bridge approaches. These results are shown in Figure 1.

The analytical modelling reported in the literature dealing with truck transportation of nuclear fuel shipping containers examined only crash situations/19,20/.

A summary of the work performed by three US Laboratories dealing with shock environments for large shipping containers during rail coupling operations is presented in the following.

Sandia Laboratories published results from tests which had been conducted in the early 1960's using ATMX cars as well as a spring-mass model developed to simulate a representative spent fuel cask system and the ATMX car/2,3,4,5/. Sandia tests were conducted at coupling speeds of 8.45, 10.78, 14.21 and 17.78 km/h with a relatively light cargo (approximately

10 000 lbs - 44 500 N. Compilation of data on observed impact velocities during rail coupling showed that 63.5% of the observed coupling velocities were below 8.45 km/h, nearly 90% were at or below 10.78 km/h, 98% were below 14.21 km/h, and 99.8% were below 17.78 km/h.

The response spectra for the ATMX car tests performed by Sandia at 17.78 km/h impact speed are shown in Figures 2 and 3 for the vertical and longitudinal axes respectively. These data combine to define the response at the interface between the cargo and the cargo floor. The analytical model developed by Sandia is shown in Figure 4. As seen in the figure, the railcar model was divided into seven masses which were determined by the car structure. The cask was represented by a single mass  $M_c$ . A separate mass of 801 000 N (180 000 lbs) was used to represent a second loaded car with which the model car was coupled. This model was used in a computer program to produce mathematically the shock descriptions in terms of acceleration versus time at the interface between the cargo and the carrier floor. Shock response spectra were developed from the shock description at this interface. A representative sample of the analytical results obtained by Sandia is shown in Figures 5 and 6.

The major conclusions reached in this study for 17.78 km/h coupling velocity are:

- (a) Containers should be tied securely to the railcar to avoid any relative motion between the cargo and the car.
- (b) For railcars equipped with standard draft gear, cargo response decreased as weight increased up to the point where the coupler springs bottom out. This bottoming caused increased response in the tie-down structure.
- (c) If the cargo is tied securely to the railcar and shock attenuating couplers are used, cargo weight has little effect on the cargo response to coupling impacts.

Los Alamos Scientific Laboratory performed an analytical study in which the loads produced during coupling of railcars carrying heavy shipping containers were investigated/6/. The structural model of the impact event was represented by a lumped-parameter technique. Figure 7 shows a side-view schematic of a specific railcar and coupler, a shipping container with its tiedown configuration, and the mass lumping schematic with degrees of freedom assigned to each lumped mass as defined in /6/. Mass 11 represents the initially stationary train that the railcar strikes. The container geometry is based on the NFS-M100 cask system. The mass lumps 1-7 represent the moving railcar, and mass lumps 8-10 represent the shipping container.

Figure 8 shows the variation in peak acceleration with the container weight for the variables of interest as reported in /6/. The significant result shown in this figure is that, for heavy containers, the vertical acceleration of the railcar is nearly as large as the horizontal deceleration which emphasizes the importance of vertical dynamics.

Figure 9 shows the variation of peak acceleration with impact speed during railcar coupling. The variation of peak acceleration with tie down axial stiffness is shown in Figure 10 for an impact speed of 17.7 km/h. The main feature of the Los Alamos study was the emphasis on vertical motion and bending. The study concluded that these are important effects to be considered in the design of tiedown structures.

A computer model, CARDS (Cask-Railcar Dynamic Simulator) was developed by the Hanford Engineering Development Laboratory for the US-NRC to provide input data for a broad range of radioactive material package-tiedown structural assessments/7,8/. CARDS is a two-dimensional, multi-degree-of-freedom model which determines the horizontal (longitudinal), vertical, and rotational motion of both the package and its transporter following impact with an anvil train during coupling operations. The model shown in Figure 11 consists of twelve equations of motion, one derived for each degree of freedom (generalized coordinate), and supplementary auxiliary equations. The equations of motion were derived from an energy balance (expressed in generalized coordinates) of the system.

Validation of the model has been established by the simulation of some 18 instrumented impact tests conducted at the Savannah River Laboratories. A comparison of selected response variables are presented in Figures 12 through 15/21/. Figure 12 shows the longitudinal force of interaction between the cask and railcar. Figure 13 presents the horizontal acceleration of the cask. Figures 14 and 15 show the vertical accelerations of the cask at the far and struck ends, respectively.

Reference /19/ reports that the most severe and therefore most damaging shocks occur during switching operations in the railroad classification yard. This study also reports that the longitudinal impulse occurring during coupling, using standard draft gear, in addition to direct cargo damage, tends to strain the tie-down system and shift the cargo. Also, the longitudinal shock induces an equally severe vertical shock, due primarily to the excitation of body-bending modes of vibration. Shock response spectra show magnitudes in the hundreds of g's in the higher frequency range, primarily due to the response of lightly-damped body bending modes which tend to "ring" at impact. Finally, Reference 9 also concludes that the use of

cushioned draft gear of various types (eg, hydraulic, elastometric) can reduce forces transmitted to the car body by an order of magnitude during switching impacts.

### 3.0 RANDOM RESPONSE OF A TRACTOR-SEMITRAILER SYSTEM CARRYING AN IRRADIATED NUCLEAR FUEL SHIPPING CASK

As mentioned earlier, the analytical modelling reported in the literature/19,20/ dealing with truck transportation of nuclear fuel shipping containers, examined only crash situations and hence did not examine the shock and vibration loads which will be present during normal off-site transportation. Accordingly, a mathematical model has been developed to predict the random response of a tractor - semitrailer system carrying a nuclear fuel shipping cask to road undulations. The model allows the flexibilities associated with the cask, tie-down, and transport frame to be simulated for various design configurations.

The objectives of this section are:

- 3.0.1 To determine the response of the trailer-cask-tie-down system to the random input of the road profile when the tractor-semitrailer system is travelling at constant speed.
- 3.0.2 To determine the levels of cask accelerations for various tractor speeds and road roughnesses.

### 3.1 Mathematical Modelling

#### 3.1.1 The Tractor-Semitrailer Model

The tractor-semitrailer model used in this study is shown in Figure 16. It is seen to be two dimensional in the longitudinal and vertical plane. The cab, engine and tractor chassis are considered as one rigid body as is the semitrailer. Both units are allowed to translate in the forward and vertical directions and to pitch, except as constrained by the fifth wheel. The unsprung mass of each wheel and axle assembly is represented by a mass having vertical translational freedom only. The tractor-semitrailer suspension is represented by linear springs and linear viscous dampers. Each of the tires on the tractor and semitrailer is assumed to be a linear spring and damper having point contact with the road surface.

By considering the constraints imposed by the fifth wheel on the motion of the tractor and the semitrailer, the model includes six degrees of freedom: pitching and bouncing motions of the tractor center of gravity, pitching motion of the semitrailer center of gravity, and vertical motion of each wheel and axle assembly.

The Lagrangian energy method was used to derive the six simultaneous differential equations of motion. The following are expressions of the energy terms:

$$T = \frac{1}{2} \dot{\bar{x}}^T M \dot{\bar{x}} \quad (1)$$

$$U = \frac{1}{2} \bar{x}^T K \bar{x} \quad (2)$$

$$D = \frac{1}{2} \dot{\bar{x}}^T C \dot{\bar{x}} \quad (3)$$

where  $T$ ,  $U$ , and  $D$  are the kinetic, potential and dissipative energy terms respectively.  
 $M$  is a mass matrix  
 $K$  is a stiffness matrix  
 $C$  is a linear (viscous) damping matrix  
 $\bar{x}$  is a set of independent generalized coordinates.

The equations of motion of the tractor-semitrailer model are given in Appendix I.

The Lagrange equations of motion are:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\bar{x}}_j} \right) + \frac{\partial D}{\partial \dot{\bar{x}}_j} - \frac{\partial U}{\partial \bar{x}_j} = 0 \quad (4)$$

Equations 1 to 4 yield a set of simultaneous, nonhomogenous, second order linear differential equations with constant coefficients.

### 3.1.2 The Cask-Tiedown-Transport Frame Model

The cask/tiedown/transport frame model is shown in Figure 17. It is seen to be a three dimensional rectangular plate finite elements allowing both in-plane and out-of-plane (bending) forces. Elements 1 to 8 represent the transport frame and the bottom plate of the cask, elements 9, 10, 15, and 16 the side walls of the cask, elements 11 to 14 the cask lid, while elements 17 to 20 represent the front and back walls of the cask as well as the tiedown system.

The rectangular plate element allowing in-plane forces is shown in Figure 18. Each node has two degrees of freedom and the

forces acting on the nodes lie in the local x-y plane. The element stiffness matrix  $[K_e]$  in local coordinates is obtained by solving the plate's differential Equations /14/ and is given by equation(5).

The mass matrix  $[M_e]_{\text{in-plane}}$  is given by /2/:

$$[M_e]_{\text{in-plane}} = \frac{\rho V}{36} \begin{bmatrix} 4 & 2 & 1 & 2 & | & & & \\ 2 & 4 & 2 & 1 & | & & & 0 \\ 1 & 2 & 4 & 2 & | & & & \\ 2 & 1 & 2 & 4 & | & & & \\ \hline & & & & | & 4 & 2 & 1 & 2 \\ & 0 & & & | & 2 & 4 & 2 & 1 \\ & & & & | & 1 & 2 & 4 & 2 \\ & & & & | & 2 & 1 & 2 & 4 \end{bmatrix} \quad (6)$$

where  $\rho$  is the mass density per unit volume and,  
V is the volume

Figure 19 shows the rectangular plate element allowing out-of-plane (bending) forces. The element stiffness matrix  $[K_e]_{\text{out-of-plane}}$  is given by /2/:

$$[K_e]_{\text{out-of-plane}} = \begin{bmatrix} K_{I,I} & | & \text{Symmetric} \\ \hline K_{II,I} & | & K_{II,II} \end{bmatrix} \quad (7)$$

where the submatrices  $K_{I,I}$ ,  $K_{II,I}$ , and  $K_{II,II}$  are presented in the following:

Equation

|                                             |   | 1                           | 2                           | 3                           | 4                           | 5                           | 6                           | 7                           | 8                           |
|---------------------------------------------|---|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $(K_e)_{in-plane} = \frac{Et}{12(1-\nu^2)}$ | 1 | $4\beta+2(1-\nu)\beta^{-1}$ |                             |                             |                             |                             |                             |                             |                             |
|                                             | 2 | $\frac{3}{2}(1+\nu)$        | $4\beta^{-1}+2(1-\nu)\beta$ |                             |                             |                             |                             |                             |                             |
|                                             | 3 | $2\beta-2(1-\nu)\beta^{-1}$ | $-\frac{3}{2}(1-3\nu)$      | $4\beta+2(1-\nu)\beta^{-1}$ |                             |                             |                             |                             |                             |
|                                             | 4 | $\frac{3}{2}(1-3\nu)$       | $-4\beta^{-1}+(1-\nu)\beta$ | $-\frac{3}{2}(1+\nu)$       | $4\beta^{-1}+2(1-\nu)\beta$ |                             |                             |                             |                             |
|                                             | 5 | $-2\beta-(1-\nu)\beta^{-1}$ | $-\frac{3}{2}(1+\nu)$       | $-4\beta+(1-\nu)\beta^{-1}$ | $-\frac{3}{2}(1-3\nu)$      | $4\beta+2(1-\nu)\beta^{-1}$ |                             |                             |                             |
|                                             | 6 | $-\frac{3}{2}(1+\nu)$       | $-2\beta^{-1}-(1-\nu)\beta$ | $\frac{3}{2}(1-3\nu)$       | $2\beta^{-1}-2(1-\nu)\beta$ | $\frac{3}{2}(1+\nu)$        | $4\beta^{-1}+2(1-\nu)\beta$ |                             |                             |
|                                             | 7 | $-4\beta+(1-\nu)\beta^{-1}$ | $\frac{3}{2}(1-3\nu)$       | $-2\beta-(1-\nu)\beta^{-1}$ | $\frac{3}{2}(1+\nu)$        | $2\beta-2(1-\nu)\beta^{-1}$ | $-\frac{3}{2}(1-3\nu)$      | $4\beta+2(1-\nu)\beta^{-1}$ |                             |
|                                             | 8 | $-\frac{3}{2}(1-3\nu)$      | $2\beta^{-1}-2(1-\nu)\beta$ | $\frac{3}{2}(1+\nu)$        | $-2\beta^{-1}-(1-\nu)\beta$ | $\frac{3}{2}(1-3\nu)$       | $-4\beta^{-1}+(1-\nu)\beta$ | $-\frac{3}{2}(1+\nu)$       | $4\beta^{-1}+2(1-\nu)\beta$ |

Symmetric

(5)

where  $E$  = Young's modulus  
 $t$  = plate thickness  
 $\nu$  = Poisson's ratio  
 $\beta = \frac{b}{a}$



Equation

|                                                  |                                                 |                                              |                                                  |                                                  |                                              |
|--------------------------------------------------|-------------------------------------------------|----------------------------------------------|--------------------------------------------------|--------------------------------------------------|----------------------------------------------|
| $-2(\beta^2 + \beta^{-2}) + \frac{1}{5}(14-4v)$  | $-\beta^{-2} + \frac{1}{5}(1-v) b$              | $\beta^2 - \frac{1}{5}(1-v) a$               | $-2(2\beta^2 - \beta^{-2}) - \frac{1}{5}(14-4v)$ | $-\beta^{-2} + \frac{1}{5}(1+4v) b$              | $2\beta^2 + \frac{1}{5}(1-v) a$              |
| $\beta^{-2} - \frac{1}{5}(1-v) b$                | $\frac{1}{3}\beta^{-2} + \frac{1}{15}(1-v) b^2$ | 0                                            | $-\beta^{-2} + \frac{1}{5}(1+4v) b$              | $\frac{2}{3}\beta^{-2} - \frac{4}{15}(1-v) b^2$  | 0                                            |
| $-\beta^2 + \frac{1}{5}(1-v) a$                  | 0                                               | $\frac{1}{3}\beta^2 + \frac{1}{15}(1-v) a^2$ | $-[2\beta^2 + \frac{1}{5}(1-v) a$                | 0                                                | $\frac{2}{3}\beta^2 - \frac{1}{15}(1-v) a^2$ |
| $-2(2\beta^2 - \beta^{-2}) - \frac{1}{5}(14-4v)$ | $-\beta^{-2} + \frac{1}{5}(1+4v) b$             | $2\beta^2 + \frac{1}{5}(1-v) a$              | $-2(\beta^2 + \beta^{-2}) + \frac{1}{5}(14-4v)$  | $-\beta^{-2} - \frac{1}{5}(1-v) b$               | $\beta^2 - \frac{1}{5}(1-v) a$               |
| $\beta^{-2} - \frac{1}{5}(1+4v) b$               | $\frac{2}{3}\beta^{-2} - \frac{4}{15}(1-v) b^2$ | 0                                            | $-\beta^{-2} + \frac{1}{5}(1-v) b$               | $-\frac{1}{3}\beta^{-2} + \frac{1}{15}(1-v) b^2$ | 0                                            |
| $-[2\beta^2 + \frac{1}{5}(1-v) a$                | 0                                               | $\frac{2}{3}\beta^2 - \frac{1}{15}(1-v) a^2$ | $-\beta^2 + \frac{1}{5}(1-v) a$                  | 0                                                | $\frac{1}{3}\beta^2 + \frac{1}{15}(1-v) a^2$ |

$$K_{11,1} = \frac{Et^3}{12(1-\nu^2)ab}$$

(9)

Equation

$$K_{11,11} = \frac{Et^3}{12(1-\nu^2)ab}$$

(10)

|                                                   |                                                    |                                                 |                                                  |                                                    |                                                 |  |  |
|---------------------------------------------------|----------------------------------------------------|-------------------------------------------------|--------------------------------------------------|----------------------------------------------------|-------------------------------------------------|--|--|
| $4(\beta^2 + \beta^{-2}) + \frac{1}{5}(14-4\nu)$  |                                                    |                                                 |                                                  |                                                    |                                                 |  |  |
| $-12\beta^{-2} + \frac{1}{5}(1+4\nu) b$           | $[\frac{4}{3}\beta^{-2} + \frac{4}{15}(1-\nu)]b^2$ |                                                 |                                                  |                                                    |                                                 |  |  |
| $[2\beta^2 + \frac{1}{5}(1+4\nu)]a$               | $-vab$                                             | $[\frac{4}{3}\beta^2 + \frac{4}{15}(1-\nu)]a^2$ |                                                  |                                                    |                                                 |  |  |
| $2(\beta^2 - 2\beta^{-2}) - \frac{1}{5}(14-4\nu)$ | $[2\beta^2 + \frac{1}{5}(1-\nu)]b$                 | $[b^2 - \frac{1}{5}(1+4\nu)]a$                  | $4(\beta^2 + \beta^{-2}) + \frac{1}{5}(14-4\nu)$ |                                                    |                                                 |  |  |
| $-12\beta^{-2} + \frac{1}{5}(1-\nu) b$            | $[\frac{2}{3}\beta^{-2} - \frac{1}{15}(1-\nu)]b^2$ | 0                                               | $[2\beta^{-2} + \frac{1}{5}(1+4\nu)]b$           | $[\frac{4}{3}\beta^{-2} + \frac{4}{15}(1-\nu)]b^2$ |                                                 |  |  |
| $[b^2 - \frac{1}{5}(1+4\nu)]a$                    | 0                                                  | $[\frac{2}{3}\beta^2 - \frac{4}{15}(1-\nu)]a^2$ | $[2\beta^2 + \frac{1}{5}(1+4\nu)]a$              | $vab$                                              | $[\frac{4}{3}\beta^2 + \frac{4}{15}(1-\nu)]a^2$ |  |  |

1 11 1

The mass matrix  $[M_e]_{\text{out-of-plane}}$  is given by /2/:

$$[M_e]_{\text{out-of-plane}} = \frac{pab}{176,400} \begin{bmatrix} 24,336 & 624b^2 & -484ab & 624a^2 & 24,336 & 624b^2 & -484ab & 624a^2 & 24,336 & 624b^2 & -484ab & 624a^2 & 24,336 & 624b^2 & -484ab & 624a^2 \\ -484ab & 624a^2 & -1,188a & -2,028b & -484ab & 624b^2 & -1,188b & -2,028a & -484ab & 624a^2 & -1,188a & -2,028b & -484ab & 624b^2 & -1,188b & -2,028a \\ 2,028b & -484ab & 286ab & 216a^2 & -3,432a & 484ab & 624a^2 & 24,336 & -3,432b & 624b^2 & 286ab & 3,432a & -484ab & 624a^2 & 286ab & 3,432b \\ -484ab & 624b^2 & 286ab & 216a^2 & -3,432a & 484ab & 624a^2 & 24,336 & -3,432b & 624b^2 & 286ab & 3,432a & -484ab & 624a^2 & 286ab & 3,432b \\ -1,188a & -286ab & 216a^2 & -3,432a & 484ab & 624a^2 & 24,336 & -3,432b & 624b^2 & 286ab & 3,432a & -484ab & 624a^2 & 286ab & 3,432b & 2,028a \\ 2,916 & 702b & -702a & 8,424 & -1,188b & -2,028a & 24,336 & -3,432b & 624b^2 & 286ab & 3,432a & -484ab & 624a^2 & 286ab & 3,432b & 2,028a \\ -702b & -162b^2 & 169ab & -1,188b & 216b^2 & 286ab & -3,432b & 624b^2 & 286ab & 3,432a & -484ab & 624a^2 & 286ab & 3,432b & 2,028a & 2,916 \\ 702a & 169ab & -162a^2 & 2,028a & -702b & -702a & 8,424 & -2,028b & 1,188a & 24,336 & 624b^2 & 286ab & 3,432a & 484ab & 624a^2 & 2,028b \\ 8,424 & 1,188b & -2,028a & 2,916 & -702b & -702a & 8,424 & -2,028b & 1,188a & 24,336 & 624b^2 & 286ab & 3,432a & 484ab & 624a^2 & 2,028b \\ 1,188b & 216b^2 & -286ab & 702b & -162b^2 & -169ab & -2,028a & 1,188a & -286ab & 216a^2 & 3,432a & 484ab & 624a^2 & 2,028b & 2,916 & 702a \\ 2,028a & 286ab & -484a^2 & 702a & -169ab & -162a^2 & 1,188a & -286ab & 216a^2 & 3,432a & 484ab & 624a^2 & 2,028b & 2,916 & 702a & 8,424 \end{bmatrix} \quad (11)$$

The two plate elements are combined to form the total plate element [in-plane forces + bending forces] as follows:

$$[K_e] = \begin{bmatrix} [K_e]_{\text{in-plane}} & 0 \\ 0 & [K_e]_{\text{out-of-plane}} \end{bmatrix} \quad (12)$$

Similarly:

$$[M_e] = \begin{bmatrix} [M_e]_{\text{in-plane}} & 0 \\ 0 & [M_e]_{\text{out-of-plane}} \end{bmatrix} \quad (13)$$

The damping matrix  $[C_e]$  was formulated by assuming proportional damping, accordingly:

$$[C_e] = \alpha [K_e] + \beta [M_e] \quad (14)$$

A transformation of the local coordinate system,  $(x, y, z)$  to the global axis system,  $(\bar{x}, \bar{y}, \bar{z})$ , is required for all element stiffness, mass, and damping matrices/18/ as follows:

$$[K_g] = [T]^T \cdot [K_e] \cdot [T] \quad (15)$$

$$[M_g] = [T]^T \cdot [M_e] \cdot [T] \quad (16)$$

$$[C_g] = [T]^T \cdot [C_e] \cdot [T] \quad (17)$$

where  $[K_g]$ ,  $[M_g]$  and  $[C_g]$  are the element stiffness, mass, and damping matrices respectively with respect to global axes.  $[T]$  is the transformation matrix of the same order as that of the element matrices:

$$[T] = \begin{bmatrix} L & 0 & 0 \dots \\ 0 & L & 0 \\ 0 & 0 & L \\ \vdots & & \end{bmatrix} \quad (18)$$

$$\text{and } [L] = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \quad (19)$$

with  $\lambda$  being a matrix of direction cosines of angles formed between the two sets of axes, ie,

$$\lambda = \begin{bmatrix} \lambda_{xx} & \lambda_{xy} & \lambda_{xz} \\ \lambda_{yx} & \lambda_{yy} & \lambda_{yz} \\ \lambda_{zx} & \lambda_{zy} & \lambda_{zz} \end{bmatrix} \quad (20)$$

in which  $\lambda_{xx} = \cos$  of angle between  $x$  and  $\bar{x}$  axis, etc.

The stiffness, inertia, and damping matrices with respect to the global axis for the individual elements are assembled to express the dynamical equations of the entire cask-tiedown-transport frame system.

In order to reduce the dynamic degrees of freedom (to save in computation time), the gyan reduction/17/ is used.

If the governing equations of motion for the plate element model are expressed as:

$$[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{x\} = \{f(t)\} \quad (21)$$

where  $[M]$ ,  $[C]$ ,  $[K]$  are the mass, damping, and stiffness matrices respectively, and  $f(t)$  is the forcing function, then Equation (21) can be written in partition form as follows:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{12}^T & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix} \quad (22)$$

where  $\{x_1\}$  represents the vector of nodal displacements which are to be retained and  $\{x_2\}$  is the corresponding vector of displacements which are to be eliminated from the system of equations.

Guyan reduction assumes that Equation (22) is to be solved subject to the constraint;

$$\{x_2\} = -[K_{22}]^{-1} [K_{12}]^T \{x_1\} = [H_{12}] \{x_1\} \quad (23)$$

which results in the following set of equations:

$$[M_C] \{\ddot{x}_1\} + [C_C] \{\dot{x}_1\} + [K_C] \{x_1\} = \{f_C\} \quad (24)$$

where:

$$[M_C] = [M_{11} + M_{12} H_{12} + H_{12}^T M_{12}^T + H_{12}^T M_{22} H_{12}] \quad (25)$$

$$[C_C] = [C_{11} + C_{12} H_{12} + H_{12}^T C_{12}^T + H_{12}^T C_{22} H_{12}] \quad (26)$$

$$[K_C] = [K_{11} + K_{12} H_{12}] \quad (27)$$

$$\{f_C\} = \{f_1(t)\} + [H_{12}]^T \{f_2(t)\} \quad (28)$$

and  $[M_C]$ ,  $[C_C]$ ,  $[K_C]$  are the condensed mass, damping, and stiffness matrices respectively.  $\{f_C\}$  is the reduced forcing vector.

The modules stack is next considered as a rigid mass having three degrees of freedom  $x_M$ ,  $y_M$ , and  $\theta_M$  (nodal point 24) connected through two beams, to nodal points 20 and 8, representing the connection between the modules and the cask, as seen in Figure 20.

The mass of the module stack (total weight 5000 kg, 11 000 lbs) was represented by:

$$[M_{\text{module}}] = \begin{bmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & m_\theta \end{bmatrix} \quad (29)$$

$[M_{\text{module}}]$  was coupled to  $[M_c]$  to form  $[M_{cc}]$  as follows:

$$[M_{cc}] = \begin{bmatrix} [M_c] & | & 0 \\ \hline 0 & | & [M_{\text{module}}] \end{bmatrix} \quad (30)$$

where  $[M_{cc}]$  is the mass matrix for the cask, tiedown, transport frame and module system.

The stiffness matrix for each of the two beams connecting nodal points 24-20 and 24-8 is generated as follows:

$$[K_{\text{module}}] = \begin{bmatrix} k_x & 0 & 0 & | & -k_x & 0 & 0 \\ 0 & k_y & 0 & | & 0 & -k_y & 0 \\ 0 & 0 & k_\theta & | & 0 & 0 & -k_\theta \\ \hline -k_x & 0 & 0 & | & k_x & 0 & 0 \\ 0 & -k_y & 0 & | & 0 & k_y & 0 \\ 0 & 0 & -k_\theta & | & 0 & 0 & k_\theta \end{bmatrix} \quad (31)$$

where  $k_x$ ,  $k_y$ , and  $k_\theta$  represent the modules stiffnesses in x, y, and  $\theta$  directions respectively.

The damping matrix  $[C_{\text{module}}]$  is generated assuming proportional damping.

In addition:

$$[K_{cc}] = \begin{bmatrix} [K_c] & | & 0 \\ \hline 0 & | & [K_{\text{module}}] \end{bmatrix} \quad (32)$$

$$[C_{cc}] = \begin{bmatrix} [C_c] & 0 \\ 0 & [C_{module}] \end{bmatrix} \quad (33)$$

Coupling Between the Cask/Tiedown/Transport Frame/Module System and the Tractor-Semitrailer Model

Figure 21 shows the complete system model. As seen, the cask/tiedown/transport frame/module system is coupled with the tractor-semitrailer model through two springs of stiffness  $K$  each, representing in reality four bolts used to fasten the transport frame to the trailer bed (see Figure 21). The stiffness  $K$  (representing two bolts) is obtained as follows:

$$K = 2 \frac{AE}{L} \quad (34)$$

where  $A$  is the bolt cross sectional area  
 $L$  is the bolt length  
 $E$  is Young's modulus of elasticity.

The moment "M" around the trailer C.G is:

$$M = 2K l_s^2 \theta_s - K l_s Y_{14} + K l_s Y_2 \quad (35)$$

where  $Y_{14}$  is the vertical displacement at nodal point 14  
and  $Y_2$  is the vertical displacement at nodal point 2

Let  $[M_{system}]$ ,  $[K_{system}]$ , and  $[C_{system}]$  be the mass, stiffness, and damping matrices of the complete system shown in Figure 21 respectively, then:

$$[M_{system}] = \begin{bmatrix} [M_{Tractor-Semitrailer}] & 0 \\ 0 & [M_{cc}] \end{bmatrix} \quad (36)$$

$$[K_{system}] = \begin{bmatrix} [K_{Tractor-Semitrailer}] & 0 \\ 0 & [K_{cc}] \end{bmatrix} \quad (37)$$

$$[C_{\text{system}}] = \begin{bmatrix} [C_{\text{Tractor-Semitrailer}}] & 0 \\ 0 & [C_{\text{cc}}] \end{bmatrix} \quad (38)$$

The coupling effect represented in Equation 35 is taken into consideration in the generation of the  $[K_{\text{system}}]$  matrix. Accordingly the quantities  $(2 K l_s^2)$ ,  $(-K l_s)$ , and  $(K l_s)$  are algebraically added to the corresponding elements in the matrix  $[K_{\text{system}}]$ .

### 3.1.3 Road Description

It has been recognized from a number of measurements on roadways that their roughness is normally distributed and can be accurately represented as a stationary random process. References 24 and 25 discuss the spectral densities that closely approximate available experimental data and propose that road roughness can be described by:

$$\begin{aligned} s(n) &= s(n_0) \left(\frac{n}{n_0}\right)^{-r_1}, \quad n \leq n_0 \\ &= s(n_0) \left(\frac{n}{n_0}\right)^{-r_2}, \quad n \geq n_0 \end{aligned} \quad (39)$$

where  $s(n)$  is the spatial density of the road roughness,  $(\text{m}^2/\text{l/m})$ .

$s(n_0)$  is the roughness coefficient (the value of the spectral density at the discontinuity frequency,  $n_0$ ).

$n$  is the spatial frequency of road roughness  
 $n_0$  is  $1/2\pi$  c/m  
 $r_1, r_2$  are waviness.

The proposed classification of the properties of road surfaces based on Equation 39 is given in Table 2 and Figure 22.

### 3.1.4 Tractor-Semitrailer System Response to Road Surface Undulations

For a  $n$ -degree-of-freedom linear system excited by a stationary random vector process,  $\bar{f}(t)$ , and having a spectral density function matrix  $s_{\bar{f}}(f)$ , and a correlation function matrix,  $R_{\bar{f}}(\tau) = E[\bar{f}^*(t_1) \bar{f}'(t_2)]$ , where  $*$  and  $'$  denote matrix

conjugation and transposition respectively,  $E[ ]$  is the expectation operator, and  $\tau = t_1 - t_2$ , we have:

$$S_{\bar{f}}(f) = \int_{-\infty}^{\infty} 2R_{\bar{f}}(\tau) \exp(-i2\pi f\tau) d\tau \quad (40)$$

$$R_{\bar{f}}(\tau) = \int_{-\infty}^{\infty} \frac{1}{2} S_{\bar{f}}(f) \exp(i2\pi f\tau) df \quad (41)$$

The impulse response function matrix,  $h(t)$ , and the corresponding frequency response function matrix,  $H(f)$ , are given by:

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(i2\pi ft) df \quad (42)$$

$$H(f) = \int_{-\infty}^{\infty} h(t) \exp(-i2\pi ft) dt \quad (43)$$

The random response,  $\bar{x}(t)$ , due to random excitation,  $\bar{f}(t)$ , is given by:

$$\bar{x}(t) = \int_{-\infty}^{\infty} h(t-\tau) \bar{f}(\tau) d\tau \quad (44)$$

The correlation function matrix,  $R_{\bar{x}}(\tau)$ , of the response is given by:

$$R_{\bar{x}}(\tau) = \frac{1}{2} \int_{-\infty}^{\infty} H^*(f) S_{\bar{f}}(f) H'(f) \exp(i2\pi f\tau) df \quad (45)$$

where  $H^*(f)$  and  $H'(f)$  are the conjugate and the transpose of  $H(f)$ , respectively. The spectral density function matrix of the response,  $S_{\bar{x}}(f)$  is given by:

$$S_{\bar{x}}(f) = H^*(f) S_{\bar{f}}(f) H'(f) \quad (46)$$

The diagonal elements of  $S_{\bar{x}}(f)$  are the spectral densities of the generalized displacements. The off-diagonal elements are the cross spectral densities of the generalized displacements. The power spectral densities for the corresponding accelerations can be determined from the relationship

$$S_{\ddot{x}}(f) = (2\pi f)^4 S_{\ddot{x}}(f) \quad (47)$$

### Input Spectral Density Matrix

Figure 23 describes the analysis of the vehicle system excited by road irregularities. The Spatial spectrum of the road roughness,  $S(n)$ , is converted into the temporal spectrum,  $S(t)$ . Taking into account the time delay caused by the wheelbase, the excitation displacement matrix of all wheelsets,  $S_w(f)$ , is obtained. This excitation displacement matrix is calculated in terms of the exciting force matrix,  $S_f(f)$ , using the forced stiffness and damping matrices.

The mathematical derivation of  $S_w(f)$  and  $S_f(f)$  is given in Appendix II.

### 3.2 Results

A digital computer program simulating the dynamics of the tractor-semitrailer/cask/tiedown system was developed. The power spectra and the rms values of the vehicle response are calculated and plotted. The effect of the speed of the vehicle on the root mean square acceleration of the system response is also investigated. The results of the computer analyses are presented in this section. These results are computed for the system parameters given in Table 3. The gross weight of the cask/tiedown/module stack is taken to be 34 545 kg (38 tonnes). Figures 24 and 25 show the configuration used in the analysis for the cask and the support frame respectively. Four road surface qualities as given in Table 2 are used. For each road condition, and assuming a vehicle forward speed of 80 km/h, the following results are obtained:

- (a) Power spectral density of the longitudinal acceleration at the tiedown connection with the trailer.
- (b) Power spectral density of the vertical acceleration at the tiedown connection with the trailer.
- (c) Power spectral density of the longitudinal acceleration at the tiedown connection with the cask.
- (d) Power spectral density of the vertical acceleration at the tiedown connection with the cask.
- (e) Power spectral density of the longitudinal acceleration of the module stack centre of gravity.

- (f) Power spectral density of the vertical acceleration of the module stack centre of gravity.

In each case the power spectral density is calculated and plotted against road frequency in Hz. Sample results are shown in Figures 26 to 30 for road Class A as defined in Table 2. Similar results are plotted in Figures 31 to 35, 36 to 40, 41 to 45 for road classes, B, C, and D respectively.

In addition to the above results which were obtained for a vehicle forward speed of 80 km/h, the effect of varying the vehicle speed on the vertical acceleration of the module stack C.G. and at the tiedown connection with the trailer was evaluated. The results are presented in Figures 50 and 51 for vehicle speeds up to 100 km/h.

#### Discussion of the Results

Figures 26 to 45 show the power spectral density at selected points in the system for a vehicle constant speed of 80 km/h. The figures as expected show that acceleration levels increase as the road surface conditions deteriorates. Best results are obtained for a smooth road, with surface class A.

The effect of the forward speed of the vehicle is investigated in Figures 46 and 47 for two selected points in the transportation system. In general the acceleration levels increase with increase in speed. Referring to Figure 46, and for a vehicle speed of 100 km/h, the RMS value of the vertical acceleration of the module C.G. for road class D is about one order of magnitude higher than the one corresponding to road class A.

The model described in this report includes the flexibilities of the cargo (cask/tiedown/transport frame). This is the main difference from the work described in Reference 15 and accordingly the acceleration levels are expected to be higher. In fact, they are about 60% higher in average. The levels reported in this study compares, however, more favourably with the results of previous field tests/26/ where an acceleration level of 0.32 g rms was measured in the speed range 80 to 90 km/h.

#### 4.0 END IMPACT RESPONSE OF A RAILCAR CARRYING AN IRRADIATED NUCLEAR FUEL SHIPPING CASK

The objective of this section is to determine the shock level to which a railcar-cask-tiedown system is subjected to during end collisions at a speed of 16 km/h (10 mph). The road cask, transportation frame and tiedown configuration as defined in Figures 24 and 25 are again used in this analysis.

#### 4.1 Mathematical Modelling

Figure 48 shows the analytical model used for a standard 100 tons capacity railcar/13/ and coupler, as well as the cask with its tiedown configuration. The model is based on the finite element technique with two dimensional beam elements used to model the vehicle/cask system. Each nodal point possesses longitudinal "u", vertical "v", and rotational "θ" degrees of freedom. The moving car is represented by ten beam elements. The tiedown system is represented by element 15. The shipping cask is represented by elements 13, 14, 16, 17, 18 and 19. Elements 11 and 21 represent the connection between the cask and the railcar. The coupler consists of one spring  $K_{11,20}$  and damper  $C_{11,20}$  as shown in the figure. A hydraulic draft gear with a travel of 254 mm/16/ was assumed in the analysis. Mass 20 represents the initially stationary train that the railcar strikes.

The dynamic equations of motion of the system in matrix notation are:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \quad (48)$$

where  $[M]$ ,  $[C]$  and  $[K]$  are the system mass, damping and stiffness matrices respectively.  $\{x\}$  and  $\{F\}$  are the displacement and force vectors respectively. The boundary conditions are:

$$\left. \begin{aligned} V_1 &= V_{11} = 0 \\ V_{20} &= \theta_{20} = 0 \end{aligned} \right\} \quad (49)$$

and initial conditions

$$\left. \begin{aligned} \dot{x}_j &= v_a \quad (j = 1, 2, \dots, 19) \\ \dot{x}_{20} &= 0 \end{aligned} \right\} \quad (50)$$

where  $v_a$  is the impact velocity in the direction shown in Figure 48.  $R_f$  is the sliding friction force of the initially stationary train being struck, and acting on nodal point 20. All other elements of  $\{F\}$  are zero.

The matrices  $[M]$  and  $[K]$  were formulated using classical finite element procedure/14/. The damping matrix  $[C]$  was formulated by assuming proportional damping in the tiedown members and linear viscous damping in the coupler.

A computer program was developed to compute the impact response at selected points within the system. The transient solution was obtained using the Hamming's modified predictor-corrector method which is a modified fourth order Runge-Kutta method. It is important to note that for the transient motion to include the contribution of all the mode shapes present in the mathematical model, the time interval in the integration scheme should be less than the lowest period of free oscillations.

#### 4.2 Results

In this section the results of the transient vibration analysis are presented. These results are computed for the set of data given by Table 4. The analysis for end impact was performed for two different combinations of draft gears; hydraulic-hydraulic and friction-hydraulic. The simulation of the two different combinations of draft gears was obtained as follows:

- (a) For the hydraulic-type draft gear, the damping force results from the viscous damping in the coupler;  $C_{11,20} \neq 0$  while  $R_F$  is neglected ( $R_F = 0$ ).
- (b) For the case of simulating the friction-hydraulic combination of draft gears, both types of damping forces were considered in the analysis; ie,  $R_F$  and  $C_{11,20} \neq 0$ .

The acceleration time profile calculated at the module C.G. for a railcar carrying 38-tonne cargo are shown in Figures 49 and 50 for the vertical and horizontal (longitudinal) directions respectively. The corresponding shock spectra are shown in Figures 51 and 52.

As seen in Figure 49, the peak level calculated in the vertical direction is about -3.5 g. Figure 50, shows a peak level of -10 g calculated in the horizontal direction. These results were obtained for the friction-hydraulic combination of draft gears.

#### Discussion of Results

The shock spectra defined in Figure 51 for the module C.G. vertical acceleration shows that the majority of the response is concentrated in the 0 to 200 Hz range with the acceleration level reaching about 100 g at approximately 170 Hz. The shock excitation in the horizontal direction is also concentrated in the 0 to 200 Hz range as presented in Figure 52. The large dc value ( $\approx 400$  g) shown in Figure 52 is due to the fact that the acceleration profile shown in Figure 50 is mostly negative. The shock spectra shown in Figures 51 and 52 will be considered next as the excitation mechanisms operating on the module stack structure as defined through modal testing.

## 5.0 MODULE STACK - MODAL ANALYSIS

The structural dynamics of two storage and transportation modules stacked one on top of the other is investigated in order to define the shock and vibration transmission through the modules.

This section presents the results of a three-dimensional modal testing of the two module stack. Ten modes of vibrations are identified in the frequency range 0 to 1000 Hz and are used to define a model data base for the module stack.

### 5.1 Measurements

The impact technique was used for the transfer function measurements. The exciter was an instrumented hammer of 5.4 kg mass. The hammer used was a PCB type, model No K291850. A PCB force transducer model No 206M06 was mounted on the hammer. The hammer force range (for 5 V output) was 22000 N, while its sensitivity was 0.23 mV/N. The hammer was equipped with a suitable tip to excite the vibration modes in the frequency range of interest (0 to 1000 Hz).

The response was measured using sensitive accelerometers (B&K Model No 4370, 10 pc/ms<sup>-2</sup>). Three responses were measured simultaneously together with the force signal and processed, on line, for the modal analysis. The data was processed and analyzed using the Genrad 2510 Micro Modal Analyzer.

### 5.2 Measurement Geometry

Figures 53 and 54 show the two storage modules filled with unirradiated CANDU fuel bundles and stacked one on top of the other. Responses were measured in three perpendicular directions (x, y, and z) giving a total of 102 response measurements. Figures 55, 56 and 57 show typical response measurement arrangement while Figure 58 shows the hammer used to excite the structure. The model used in the analysis and the coordinate system are defined in Figure 59. The measurement points are numbered 1 to 34. The excitation  $\bar{F}$  was applied at point 1, in the direction Y as shown in Figure 59.

### 5.3 Transfer Function Measurements

The transfer functions were computed as a complex function of frequency from averaged Fourier transforms of output accelerations and input force. In order to improve the signal-to-noise ratio each transfer function represents the average of eight measurements. The coherence function was also computed for each measurement point and was used as a measure of measurement quality throughout the measurement program.

Typical results are shown in Figures 60, 61 and 62 for the measurements taken at point 3 in the x, y, and z directions respectively. Each transfer function is expressed in terms of its magnitude and phase angle for frequency range 5 to 1000 Hz.

#### 5.4 Study of Transfer Functions

From examining the transfer functions obtained through the measurement program, the following ten resonances were identified as corresponding to the main modes of vibration:

- (a) 196.791 Hz
- (b) 253.803 Hz
- (c) 315.429 Hz
- (d) 393.350 Hz
- (e) 486.923 Hz
- (f) 518.046 Hz
- (g) 646.335 Hz
- (h) 742.175 Hz
- (i) 859.272 Hz
- (j) 946.819 Hz

#### 5.5 Modal Analysis

The modal analysis was performed using the SDCR MODAL-PLUS-V7.0 software package. To obtain the mode shapes of the module stack, the following technique was used. For each resonance of interest, a measurement (transfer function) where that resonance is clearly visible is selected. A mathematical expression is curve fitted on that measurement for the resonance and the nearest frequency region. The fit gives frequency, damping, amplitude, and phase for the resonance. These parameters are then used to give an estimate of amplitude and phase for every measurement. The mathematical basis for the modal analysis technique is given elsewhere (see eg, References 22 and 23).

In total, ten modes of vibration were identified in the frequency range of 0 to 1000 Hz. Figures 63 through 72 show the mode shapes obtained for the modules stack frame members.

#### 5.6 Study of Mode Shapes

The procedure mentioned above gives for each resonance the amplitude and phase at each measurement. When the geometry of the module stack is known, mode shapes may be generated and displayed so that each point of the module stack moves with the computed amplitude and phase in the given directions. For each mode shape shown in Figures 63 through 72 the corresponding natural frequency is also identified.

Figure 63 shows the mode shape at 196.791 Hz. The main response occurs due to bending of the top frame members. The mode at 253.803 Hz is shown in Figure 64 and is characterized mainly by bending in the side frame members. Bending of the middle frame members (bottom of the top module) occurs at 486.923 Hz (Figure 67), 518.046 Hz (Figure 68), 859.272 Hz (Figure 71), and 946.819 Hz (Figure 72).

## 6.0 SHOCK AND VIBRATION TRANSMISSION THROUGH MODULE STRUCTURE

This section presents the predictions of the actual response of the module tubes to simulated operating inputs. The modal data base (mode shapes and modal parameters) defined in Section 5.0 is used along with the force inputs defined in Section 3.0 and 4.0.

The principle of computation can be described by looking at the frequency response relationship

$$\frac{X_j}{F_j}(\omega) = H_{ij}(\omega) \quad (51)$$

where  $i$  and  $j$  are reference and response points respectively, and  $H_{ij}$  is the transfer function. If the frequency response  $H_{ij}$  has been measured and the force specified as a function of frequency, the response can be calculated from:

$$X_i = H_{ij}(F_j) \quad (52)$$

For a complete structure this relation can be written in matrix form:

$$\{X\} = [H] \{F\} \quad (53)$$

### 6.1 Vibration Response of Module Tubes: Road Transportation Model

The various components of the module analysis (road transportation model) are given in Figure 73. The power spectral density of the vertical acceleration of the module stack centre of gravity obtained for road class D and a vehicle forward speed of 80 km/h and defined in Figure 45 was used as an input to the module stack modal data base. This input was applied in the vertical direction simultaneously at points 19, 20, 23 and 24 as defined in Figure 63.

Figures 74 and 75 show the response calculated at points 25 and 31 respectively (see Figure 59). These responses are obtained in the vertical direction. The corresponding RMS values are 0.73 g and 0.31 g for Figures 74 and 75 respectively.

## 6.2 Shock Response of Module Tubes: Railcar End Impact Model

The various components of the module analysis (railcar and impact model) are given in Figure 76. The acceleration frequency profiles obtained for a railcar carrying 38-tonne cask and striking a 226.8-tonne stationary train at 16 km/h, as defined in Figures 51 and 52 for the vertical and longitudinal (horizontal) directions were used as excitation inputs in this case.

For the vertical direction, the excitation defined in Figure 51 was applied simultaneously at points 19, 20, 23 and 24 (see Figure 69) and the response calculated in the vertical direction at points 25 and 31. The results are shown in Figures 77 and 78.

Figures 79 and 80 show the results obtained due to the excitation defined in Figure 52. In this case the response is examined in the horizontal direction at points 25 and 31 respectively.

The acceleration frequency profiles presented in this section were calculated for the 0 to 1000 Hz frequency range. The displacement associated with frequencies higher than 1000 Hz was assumed minimal.

## 7.0 DISCUSSION OF THE RESULTS

The results of computer simulation of the module tube vibration response to road undulations showed vertical acceleration level of 7.2 m/s<sup>2</sup>rms, (0.73 g rms) predicted for the top row and 3.0 m/s<sup>2</sup>rms (0.31 g rms) for the bottom row of the top module. These levels are based on a semitrailer travelling at a speed of 80 km/h (50 mph) on a "class D" or very poor road. At the other road conditions, the vertical responses of the top tubes of the upper module are as follows:

| Road Class                        | A    | B    | C    |
|-----------------------------------|------|------|------|
| Acceleration m/s <sup>2</sup> rms | 0.59 | 1.37 | 3.53 |
| g rms                             | 0.06 | 0.14 | 0.36 |

In contrast, the effect on the acceleration level of vehicle speed on the same type of road is much less pronounced. For example, the reduction in acceleration level on a class D road

from 100 km/h to 20 km/h is only 40%. The vibration response is concentrated in the 15 to 450 Hz range (see Figures 74 and 75).

The railcar coupling analysis shown in Figures 77 to 80 indicates a shock response reaching in excess of  $980 \text{ m/s}^2$  peak (100 g peak) at high frequencies for the top row and approximately half this magnitude for the bottom row of the top module. The shock spectra shown in Figures 77 to 80 are in reasonable agreement with data published elsewhere in the literature/2,4,5,6,9/.

#### 8.0 CONCLUSIONS

- 8.1 The random response of a tractor-semitrailer system carrying an irradiated nuclear fuel shipping cask has been analytically determined. The results are in reasonable agreement with published experimental data.
- 8.2 An increase in road vehicle speed produces higher acceleration levels. Over a speed range of 20 to 100 km/h (12.5 to 62.4 mph) the maximum increase for any road is about 40%. Travel on very poor roads (class D) results in an increase of rms vertical acceleration of the cask (and internals) of about one order of magnitude in comparison with smooth roads (class A).
- 8.3 The vibration transmission through the two module stack structure during road transportation has been determined using the modal testing technique.
- 8.4 The simulation of the vibration environment during road transportation indicates that vertical preload of the module stack is not necessary.
- 8.5 The shock response to a coupling impact of a railcar carrying a road cask has been determined by analytical modelling.
- 8.6 For the railcar coupling situation, the predicted shock response spectra at the module tubes indicate that preload of the module stack may be necessary if the fuel bundle impact tests predict bundle failure for such a transient shock.

#### 9.0 RECOMMENDATIONS

- 9.1 It is recommended that the road transportation system not be operated on class D (very poor) roads at normal speeds for extended periods.

- 9.2 The design of the cask need not provide for module stack preload for road transport.
- 9.3 The design of the cask should provide for module stack preload for rail transport if the bundle impact tests/1/ predict bundle failure due to shock loading which could be experienced during railcar coupling.
- 9.4 The results presented in this report should be taken in the context of the limitations imposed by mathematical modelling. Actual field measurements on the prototype transportation system are recommended to verify the analytical modelling.

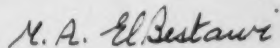
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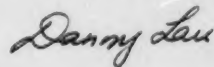
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TABLE I  
TRUCK VIBRATION/2/

|                        | Measurements on Cargo Floor (g)<br>99% Level of 0 to Peak Amplitude |                    |                  |
|------------------------|---------------------------------------------------------------------|--------------------|------------------|
| Frequency<br>Band (Hz) | Longitudinal<br>Axis                                                | Transverse<br>Axis | Vertical<br>Axis |
| 0-5                    | 0.10                                                                | 0.10               | 2.00             |
| 5-10                   | 0.08                                                                | 0.06               | 1.04             |
| 10-20                  | 0.84                                                                | 0.15               | 1.68             |
| 20-40                  | 0.51                                                                | 0.24               | 1.20             |
| 40-80                  | 0.36                                                                | 0.42               | 0.50             |
| 80-120                 | 0.24                                                                | 0.27               | 0.87             |
| 120-180                | 1.23                                                                | 0.21               | 0.63             |
| 180-240                | 0.87                                                                | 0.12               | 0.87             |
| 240-350                | 0.24                                                                | 0.15               | 0.63             |
| 350-500                | 0.24                                                                | 0.15               | 0.42             |
| 500-700                | 0.87                                                                | 0.15               | 0.87             |
| 700-1000               | 1.50                                                                | 0.87               | 1.17             |
| 1000-1400              | 0.87                                                                | 1.17               | 1.17             |
| 1400-1900              | 0.39                                                                | 0.24               | 0.87             |

TABLE 2  
ROAD SURFACE CHARACTERISTICS

| Road Class                                                        | $s(n_o)$<br>Range | $s(n_o)$<br>Geometric Mean | $r_1$ | $r_2$ |
|-------------------------------------------------------------------|-------------------|----------------------------|-------|-------|
| A Good                                                            | 8 - 32            | 16                         | 2     | 1.5   |
| B Average                                                         | 32 - 128          | 64                         | 2     | 1.5   |
| C Poor                                                            | 128 - 512         | 256                        | 2     | 1.5   |
| D Very Poor                                                       | 512 - 2048        | 1024                       | 2     | 1.5   |
| $s(n_o)$ is measured in units of $10^{-6} \text{ m}^2/\text{l/m}$ |                   |                            |       |       |

TABLE 3

BASE LINE VALUES FOR ARTICULATED VEHICLE MODEL1. TRACTORa. General

|                                   |                        |
|-----------------------------------|------------------------|
| Sprung Mass ( $M_t$ )             | 9600 kg                |
| Front unsprung mass ( $M_1$ )     | 360 kg                 |
| Rear unsprung mass ( $M_2$ )      | 1450 kg                |
| Pitch moment of inertia ( $I_t$ ) | 3390 kg·m <sup>2</sup> |

b. Dimensions

|       |        |       |        |
|-------|--------|-------|--------|
| $b_1$ | 2.25 m | $b_3$ | 1.4 m  |
| $b_2$ | 2.05 m | $a_1$ | -0.3 m |
|       |        | $a_2$ |        |

2. SEMITRAILERa. General

|                                   |                           |
|-----------------------------------|---------------------------|
| Sprung Mass ( $M_s$ )             | 9900 kg                   |
| Unsprung mass ( $M_3$ )           | 1450 kg                   |
| Pitch moment of inertia ( $I_s$ ) | 152 000 kg·m <sup>2</sup> |

b. Dimensions

|       |        |
|-------|--------|
| $b_3$ | 3.96 m |
| $b_4$ | 6.00 m |
| $a_2$ | 0.9 m  |

3. SUSPENSION CHARACTERISTICS

|       |          |       |             |
|-------|----------|-------|-------------|
| $K_1$ | 357 kN/m | $C_1$ | 11.5 kN·s/m |
| $K_2$ | 630 kN/m | $C_2$ | 29.0 kN·s/m |
| $K_3$ | 630 kN/m | $C_3$ | 29.0 kN·s/m |

4. TIRE CHARACTERISTICS

|          |           |          |            |
|----------|-----------|----------|------------|
| $K_{t1}$ | 1560 kN/m | $C_{t1}$ | 0.7 kN·s/m |
| $K_{t2}$ | 5250 kN/m | $C_{t2}$ | 1.2 kN·s/m |
| $K_{t3}$ | 5250 kN/m | $C_{t3}$ | 1.2 kN·s/m |

5. VEHICLE SPEED

80 km/h

TABLE 4

BASELINE VALUES FOR THE RAILCAR-CASK-TIEDOWN MODEL

|                                                   |                                                                                              |
|---------------------------------------------------|----------------------------------------------------------------------------------------------|
| Cask/tiedown/transport frame/module stack weight  | 38 tonnes                                                                                    |
| Railcar Weight                                    | 30 tons                                                                                      |
| Stiffness of Draft Gear $K_{11,20}$               | $= 16,462 \text{ N/m}$ (94,000 lb/in)                                                        |
| Viscous Damping in the Draft Gear $C_{11,20}$     | $= 210 \text{ 150 N/m/s}$<br>(1200 lb/in/s)                                                  |
| Stationary Train Weight                           | $= 226.8 \text{ tonnes}$ (500 000 lb)                                                        |
| Friction Force $R_F$                              | $= (\text{coefficient of friction of } 0.2) =$<br>$4.448 \times 10^5 \text{ N}$ (100 000 lb) |
| Constant of Proportional Damping $\alpha = \beta$ | $= 0.02$                                                                                     |
| $V_a$                                             | $= 16 \text{ km/h}$ (10 mph)                                                                 |
| $E$                                               | $= 2.0684 \times 10^{11} \text{ Pa}$ ( $30 \times 10^5 \text{ psi}$ )                        |
| $p$                                               | $= 0.3$                                                                                      |
| $\Delta t$                                        | $= 10 \text{ } \mu\text{s}$ (time step in the integration scheme)                            |
| Length of Railcar                                 | $= 13.182 \text{ m}$ (519 in)                                                                |

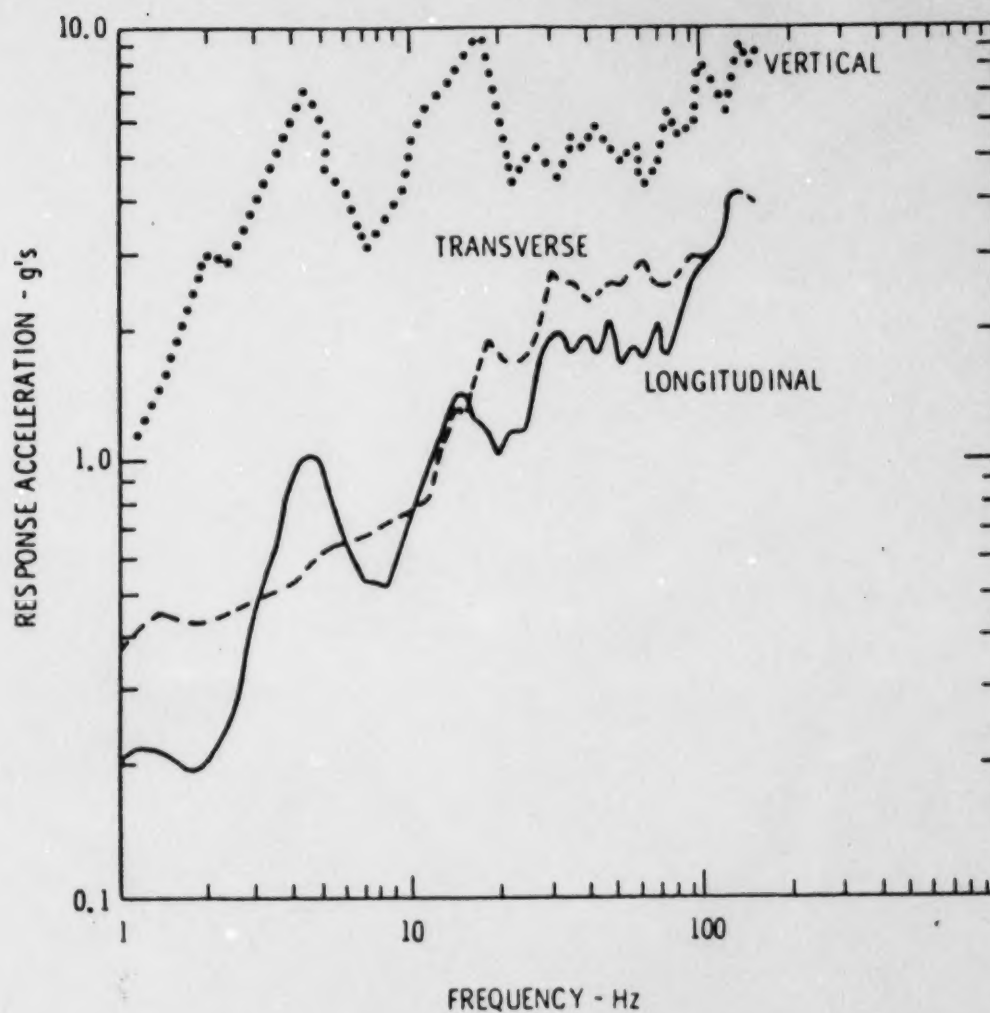


FIGURE 1  
TRUCK SUPERIMPOSED SHOCK RESPONSE ENVELOPES - 3% DAMPING

000036

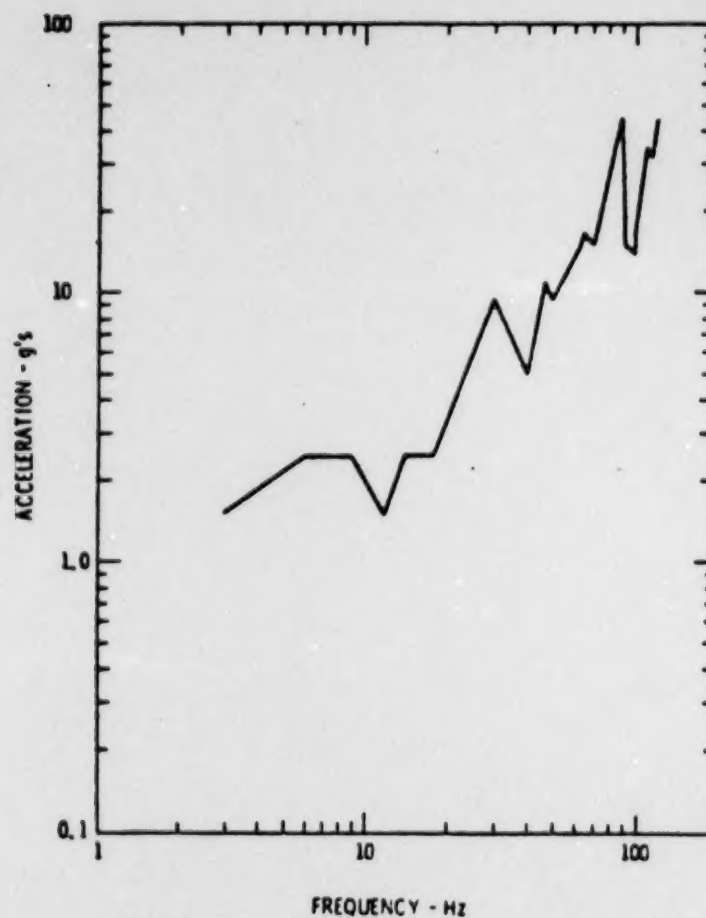


FIGURE 2

RAILROAD COUPLING SHOCK RESPONSE SPECTRUM - 3% DAMPING  
ATMX-500 - 17.78 km/hr (11.05 mph) IMPACT - VERTICAL AXIS /2/

000037

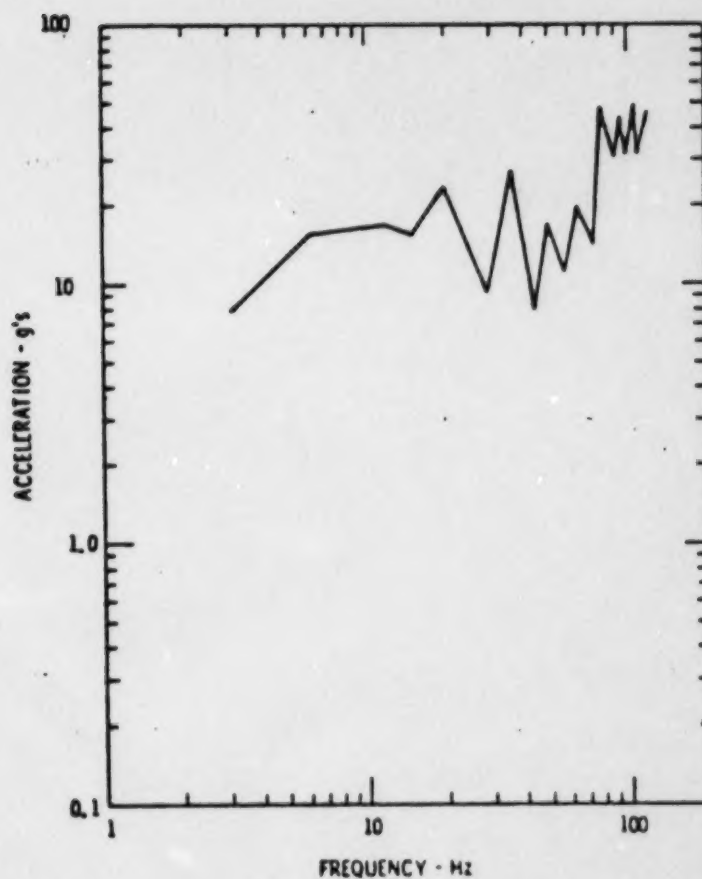
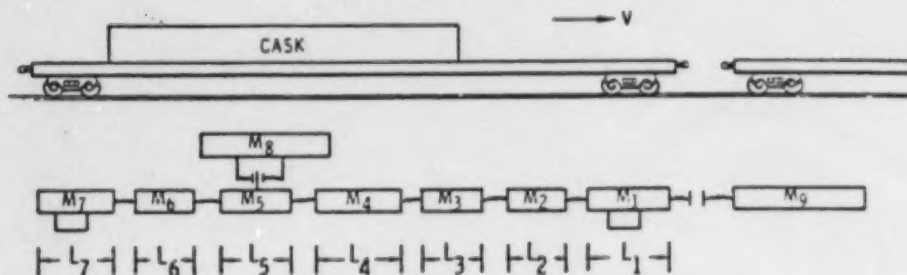


FIGURE 3  
RAILROAD COUPLING SHOCK RESPONSE SPECTRUM - 3% DAMPING  
ATMX-500 - 17.78 km/hr (11.05 mph) IMPACT - LONGITUDINAL AXIS /2/

000038



#### MODEL CHARACTERISTICS

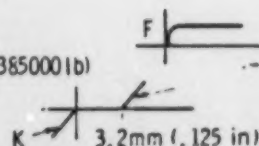
|       | WEIGHT*             |          |       | LENGTH |       |
|-------|---------------------|----------|-------|--------|-------|
|       | (N)                 | (lbs)    |       | (m)    | (in)  |
| $M_1$ | 78100               | (17553)  | $L_1$ | 2.67   | (105) |
| $M_2$ | 22600               | (5079)   | $L_2$ | 2.44   | (96)  |
| $M_3$ | 23600               | (5301)   | $L_3$ | 1.83   | (72)  |
| $M_4$ | 41100               | (9237)   | $L_4$ | 2.44   | (96)  |
| $M_5$ | 23600               | (5301)   | $L_5$ | 1.83   | (72)  |
| $M_6$ | 22600               | (5079)   | $L_6$ | 2.44   | (96)  |
| $M_7$ | 78100               | (17553)  | $L_7$ | 2.67   | (105) |
| $M_8$ | (1) 178000 (40000)  |          |       |        |       |
|       | (2) 445000 (100000) |          |       |        |       |
|       | (3) 712000 (160000) |          |       |        |       |
|       | (4) 890000 (200000) |          |       |        |       |
| $M_9$ | 801000              | (180000) |       |        |       |

#### SPRING RATES

|           |                                                                 |
|-----------|-----------------------------------------------------------------|
| $K_{1-2}$ | $8.98 \times 10^9 \text{ N/M}, 5130 \times 10^8 \text{ lb/in}$  |
| $K_{2-3}$ | $9.95 \times 10^9 \text{ N/M}, 5684 \times 10^8 \text{ lb/in}$  |
| $K_{3-4}$ | $16.21 \times 10^9 \text{ N/M}, 9256 \times 10^8 \text{ lb/in}$ |
| $K_{4-5}$ | $16.21 \times 10^9 \text{ N/M}, 9256 \times 10^8 \text{ lb/in}$ |
| $K_{5-6}$ | $9.95 \times 10^9 \text{ N/M}, 5684 \times 10^8 \text{ lb/in}$  |
| $K_{6-7}$ | $8.98 \times 10^9 \text{ N/M}, 5130 \times 10^8 \text{ lb/in}$  |

$K_{1-9}$  @ 17.78 km/hr (11.05 mph)  $F = 1712570 \text{ N} (385000 \text{ lb})$

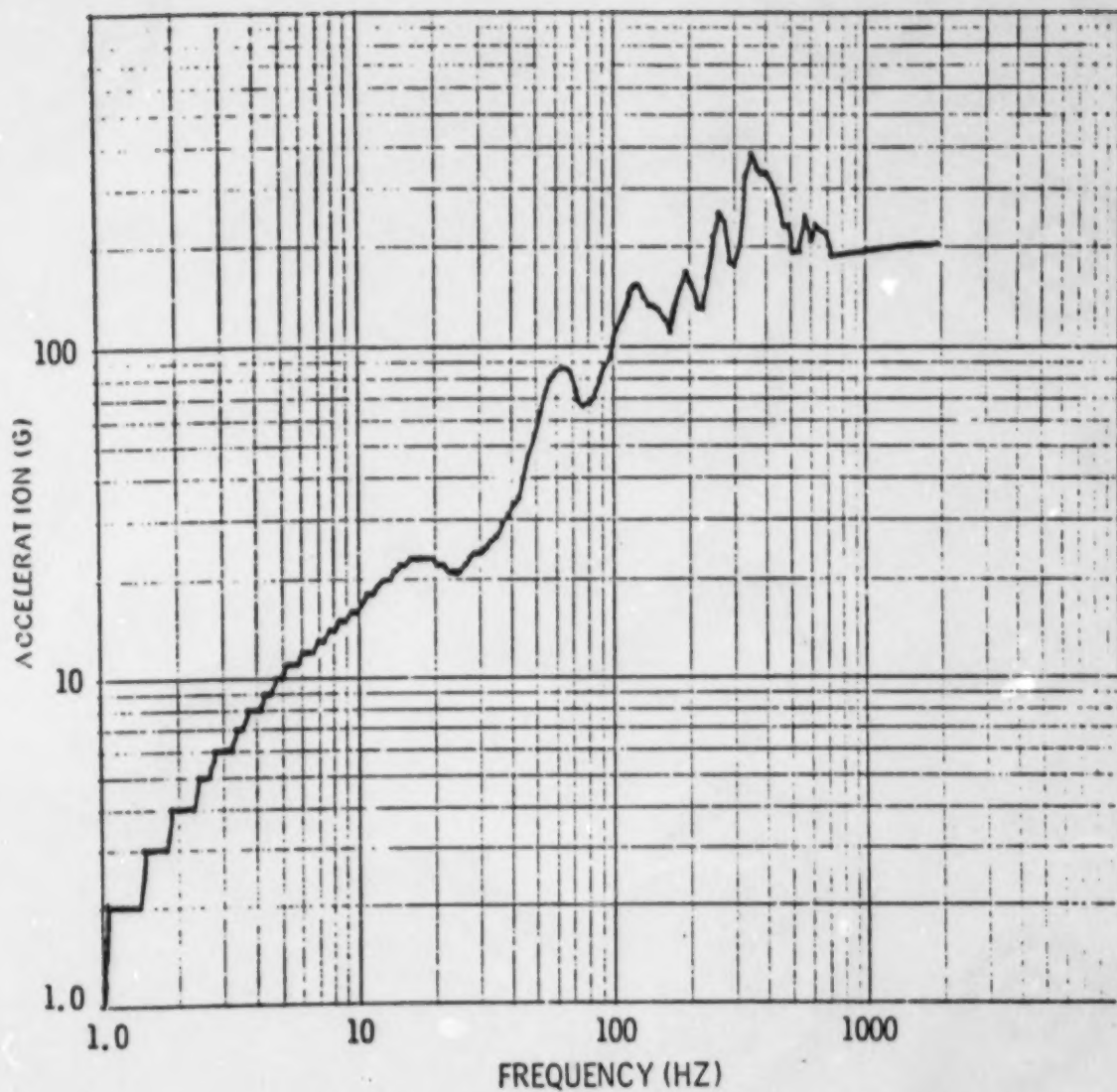
$K_{5-8} = 1.75 \times 10^9 \text{ N/M}, 1000 \times 10^8 \text{ lb/in}$



\* SEE PAGE 4, PAR. 3.4.1.2 OF ASTM METRIC PRACTICE GUIDE E380-72

FIGURE 4  
SPRING - MASS MODEL; SPENT FUEL CASK SYSTEM

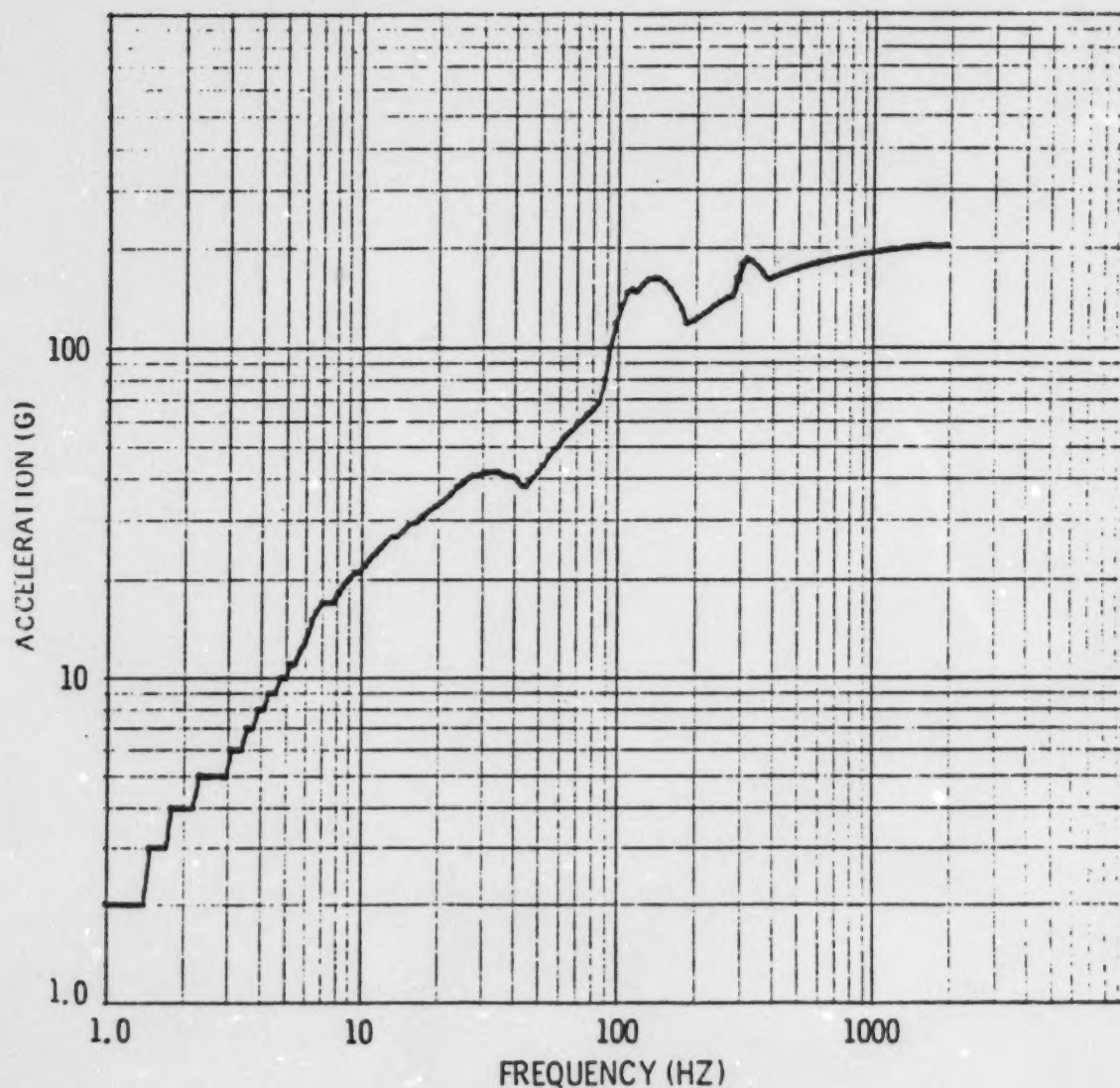
000039



Spent Fuel Cask System, 178,000 N (40,000 pound) Cargo  
17.78 km/hr (11.05 mph) Impact Velocity, 3 percent Damping,  
Longitudinal Axis

FIGURE 5  
RESPONSE SPECTRUM - ANALYTICAL RESULTS

000040



Spent Fuel Cask System, Cargo Tied Down,  
178,000 N (40,000 pound) Cargo, 17.78 km/hr  
(11.05 mph) Impact Velocity, 3 percent  
Damping, Longitudinal Axis

FIGURE 6  
RESPONSE SPECTRUM - ANALYTICAL RESULTS

000041

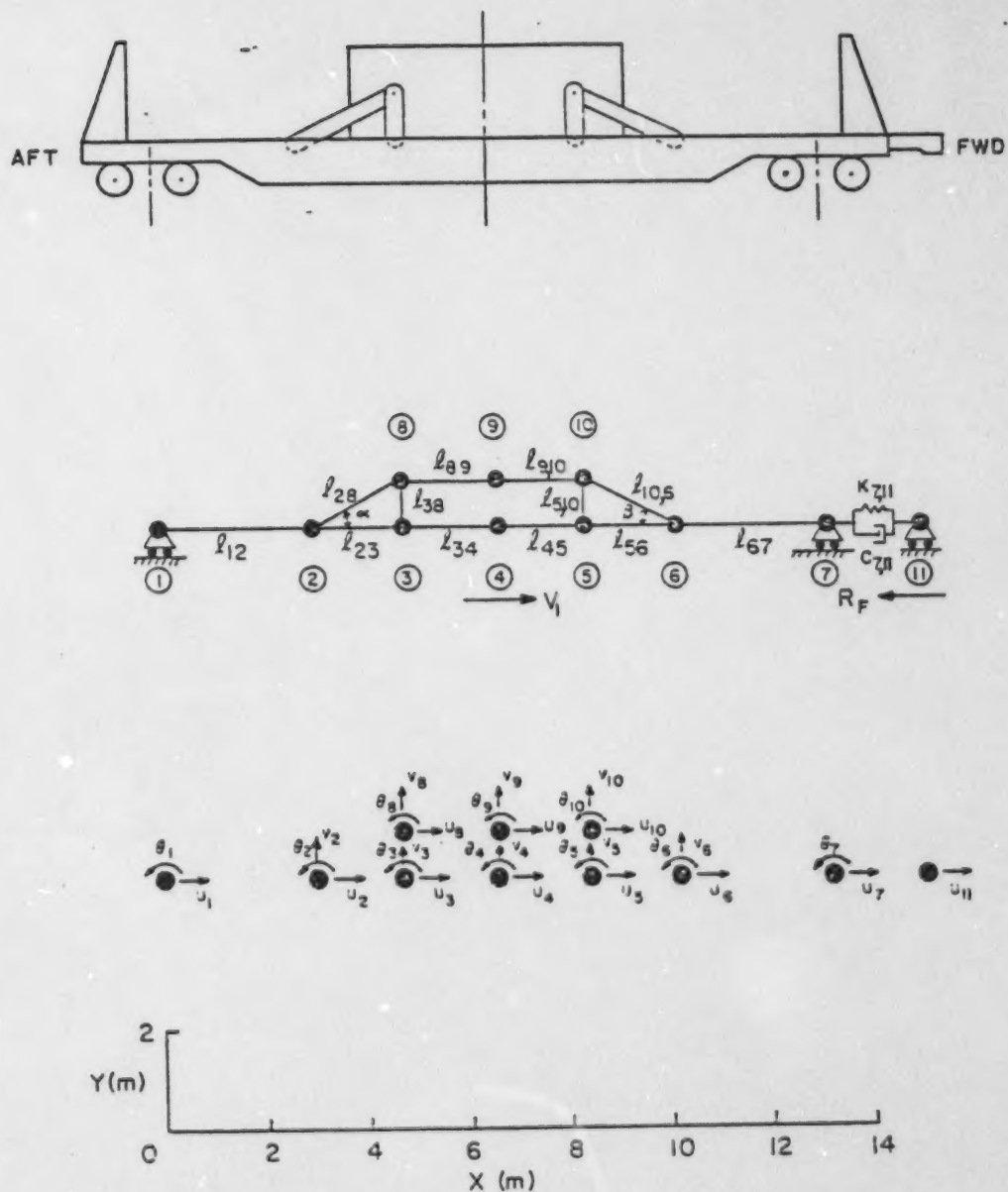


FIGURE 7  
STRUCTURAL MODELING TECHNIQUE FOR RAIL CAR,  
CONTAINER, AND COUPLER

000042

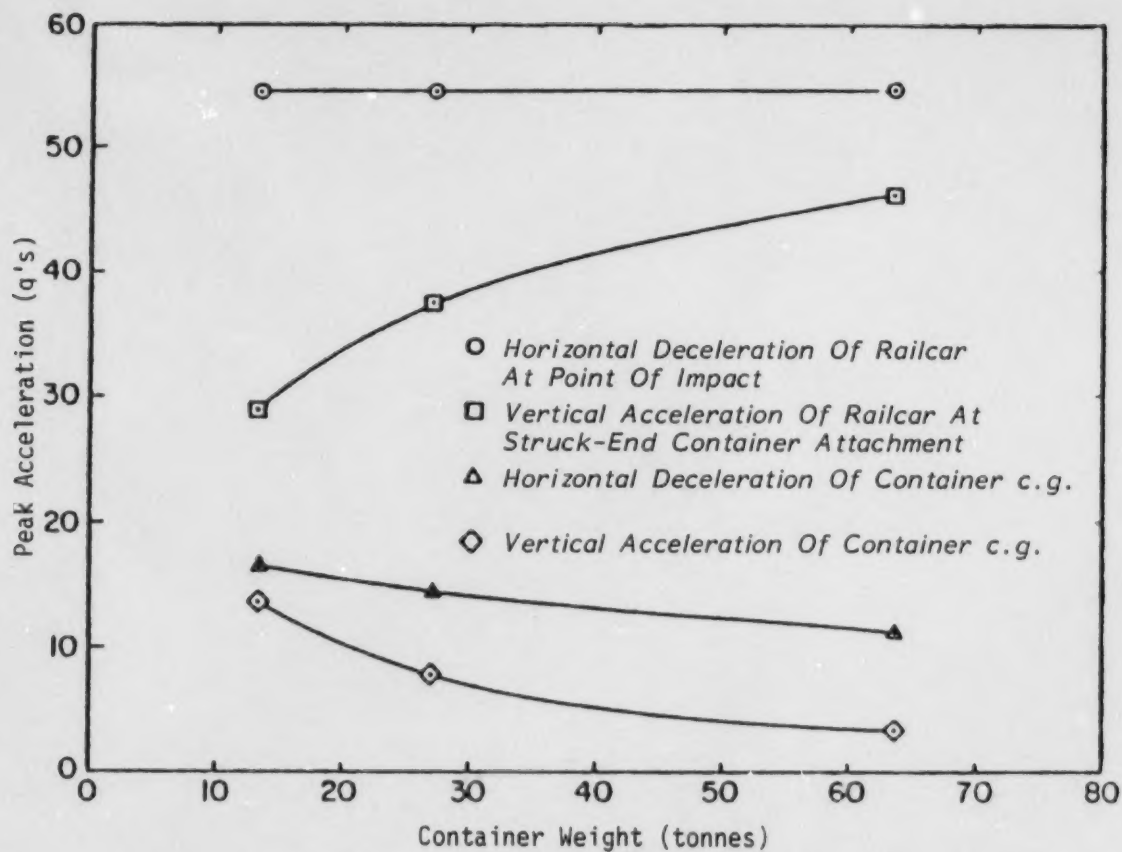
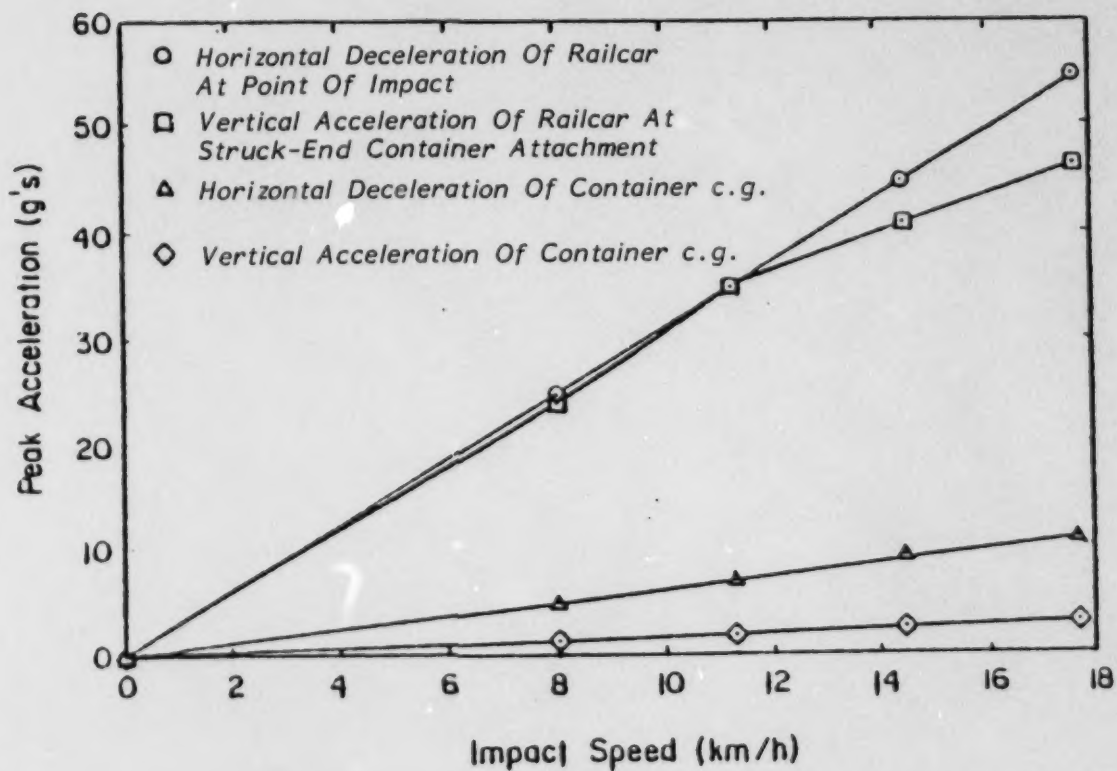


FIGURE 8  
PEAK ACCELERATION DURING IMPACT AT 17.7 km/h (11 mph)

000043



Note: Weight of stationary train =  $2.224 \times 10^6$  N (500,000 lbs).  
Resistance of stationary train =  $4.448 \times 10^5$  N (100,000 lbs).

FIGURE 9  
PEAK ACCELERATION DURING IMPACT OF RAIL CAR CARRYING  
63.5-TONNE CONTAINER

000044

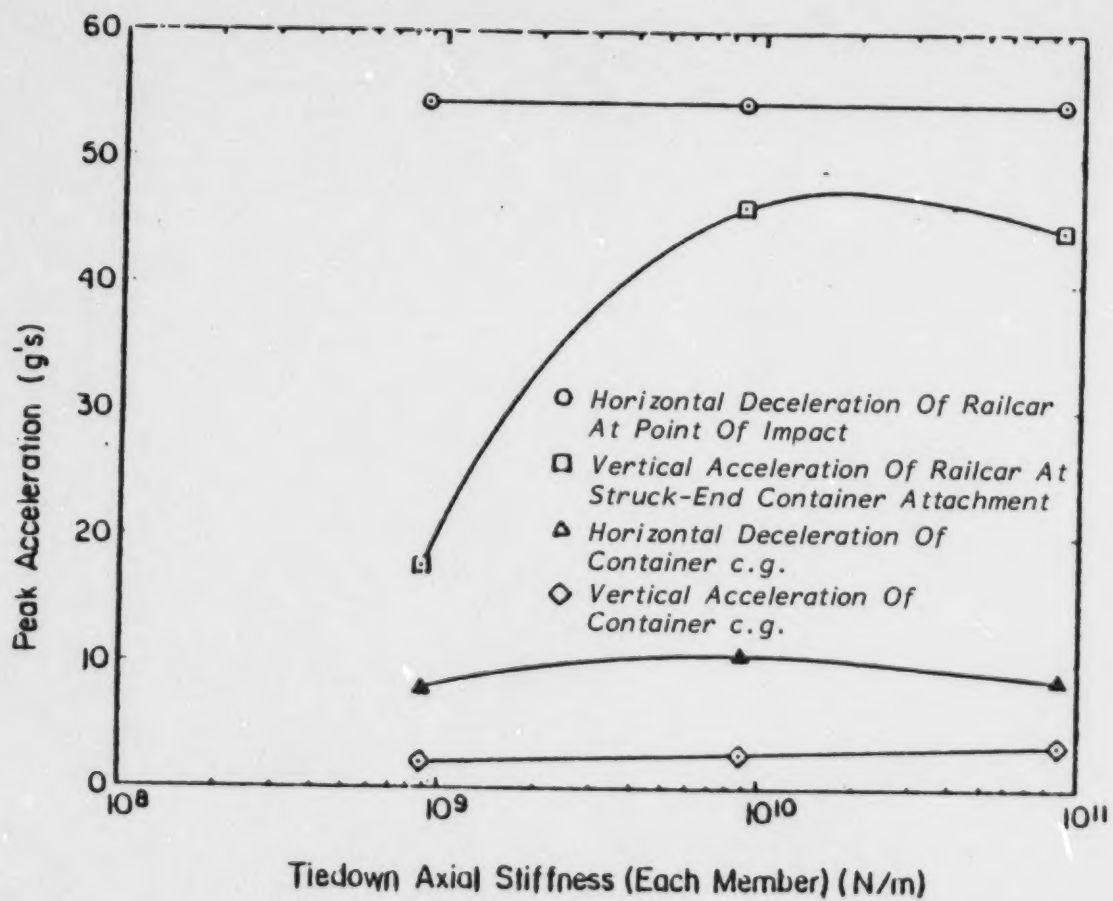


FIGURE 10  
PEAK ACCELERATION DURING IMPACT AT 17.7 km/h (11 mph)  
OF RAIL CAR CARRYING 63.5-TONNE CONTAINER

000045

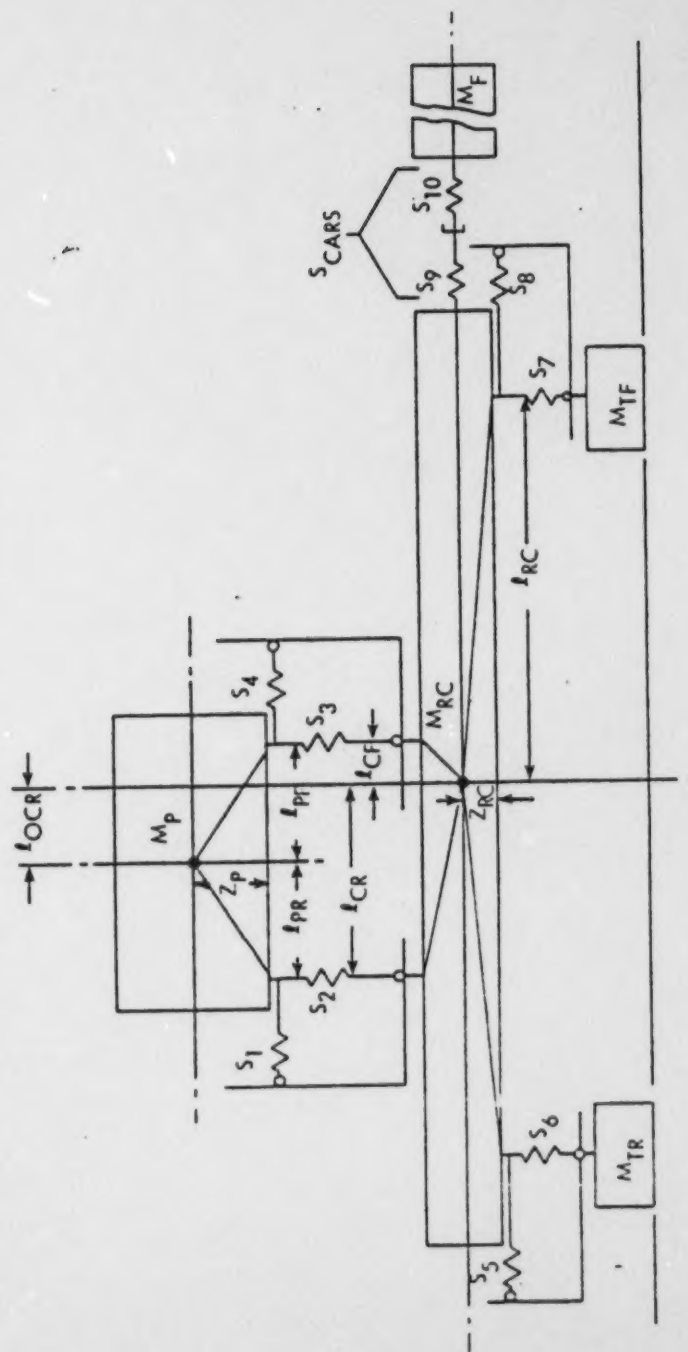


FIGURE 11  
SPRING-MASS MODEL OF CASK-RAIL CAR SYSTEM

000046

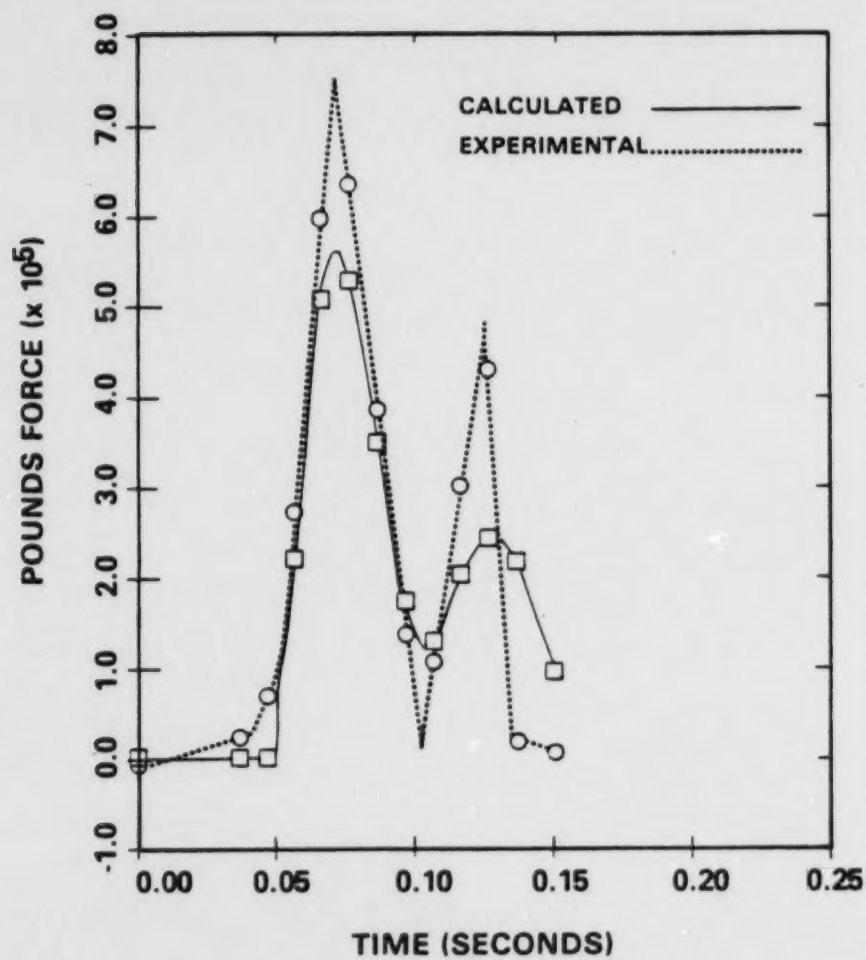


FIGURE 12  
LONGITUDINAL FORCE OF INTERACTION BETWEEN  
CASK AND RAIL CAR

000047

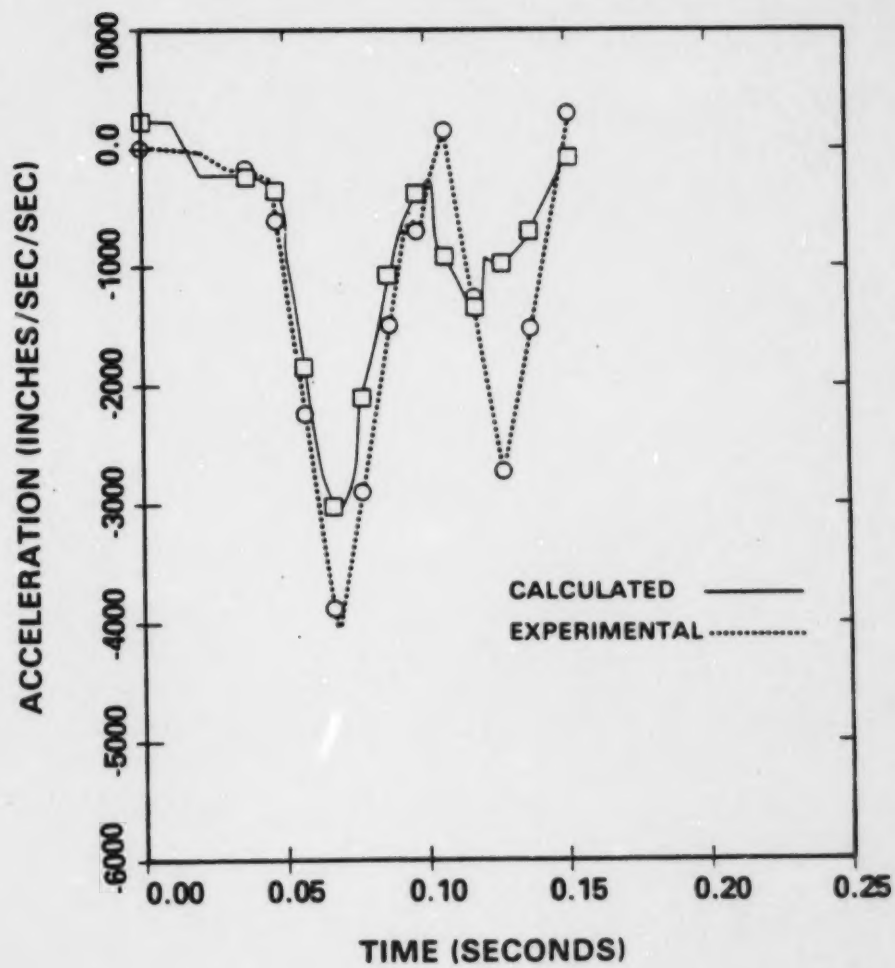


FIGURE 13  
HORIZONTAL ACCELERATION OF CASK

000048

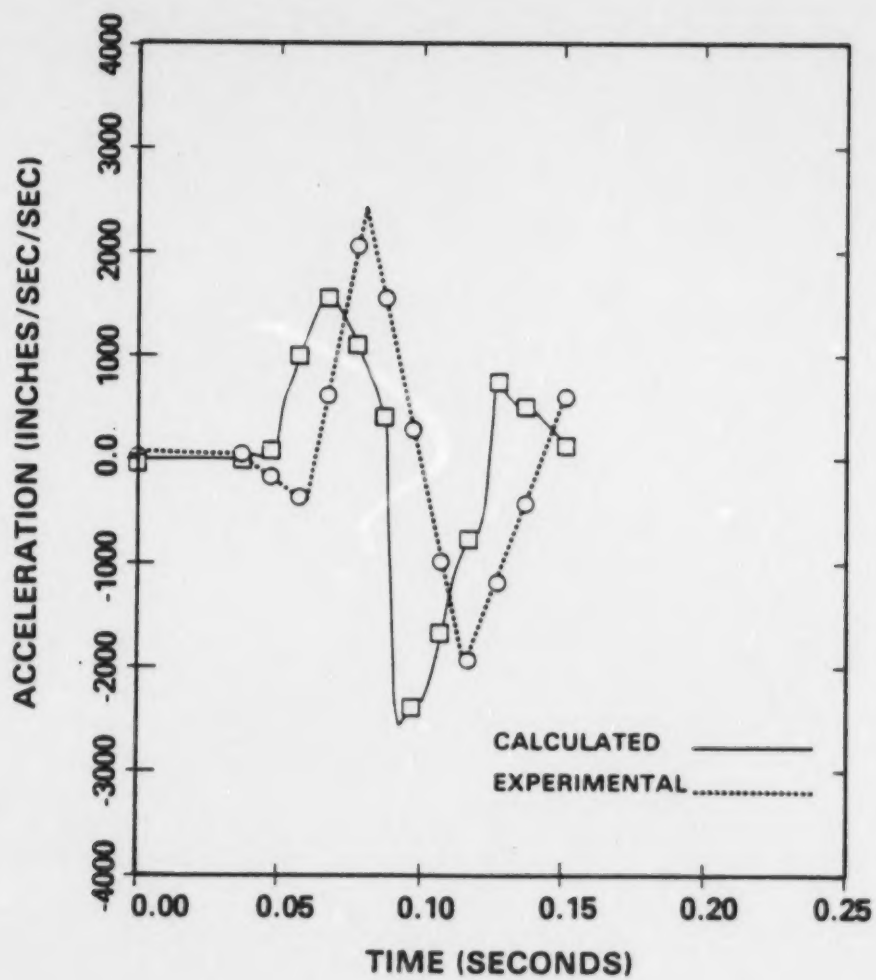


FIGURE 14  
VERTICAL ACCELERATION OF CASK AT FAR END

000049

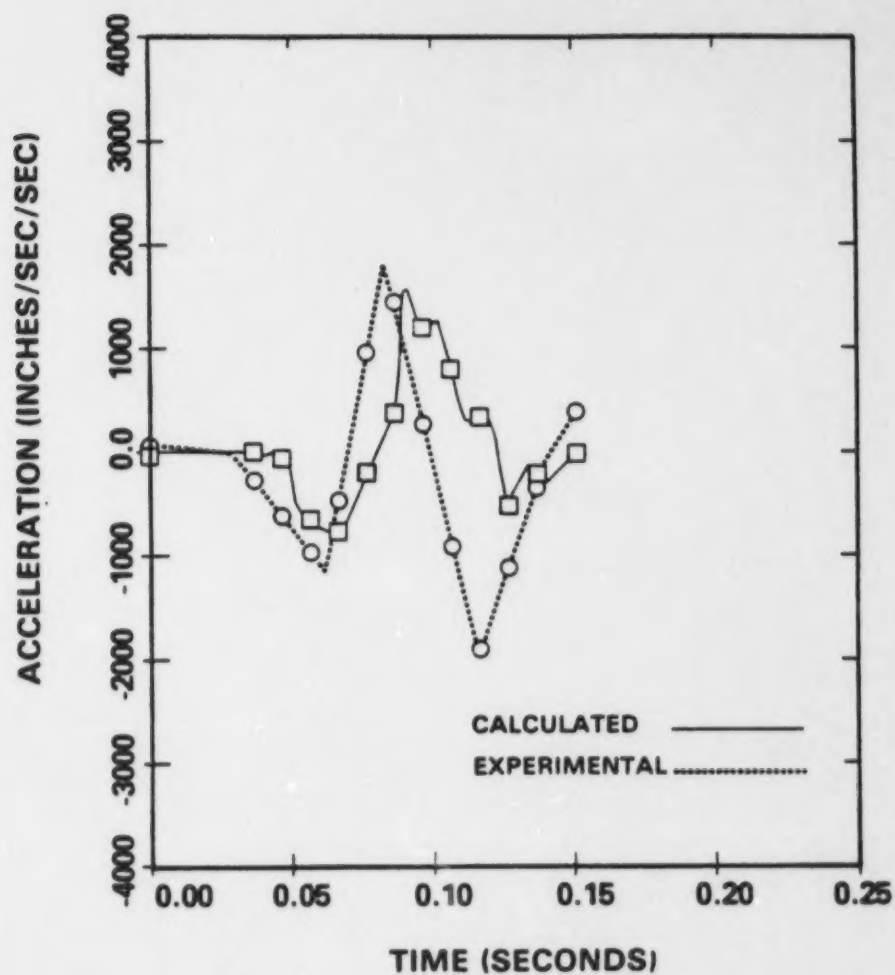


FIGURE 15  
VERTICAL ACCELERATION OF CASK AT STRUCK END

000050

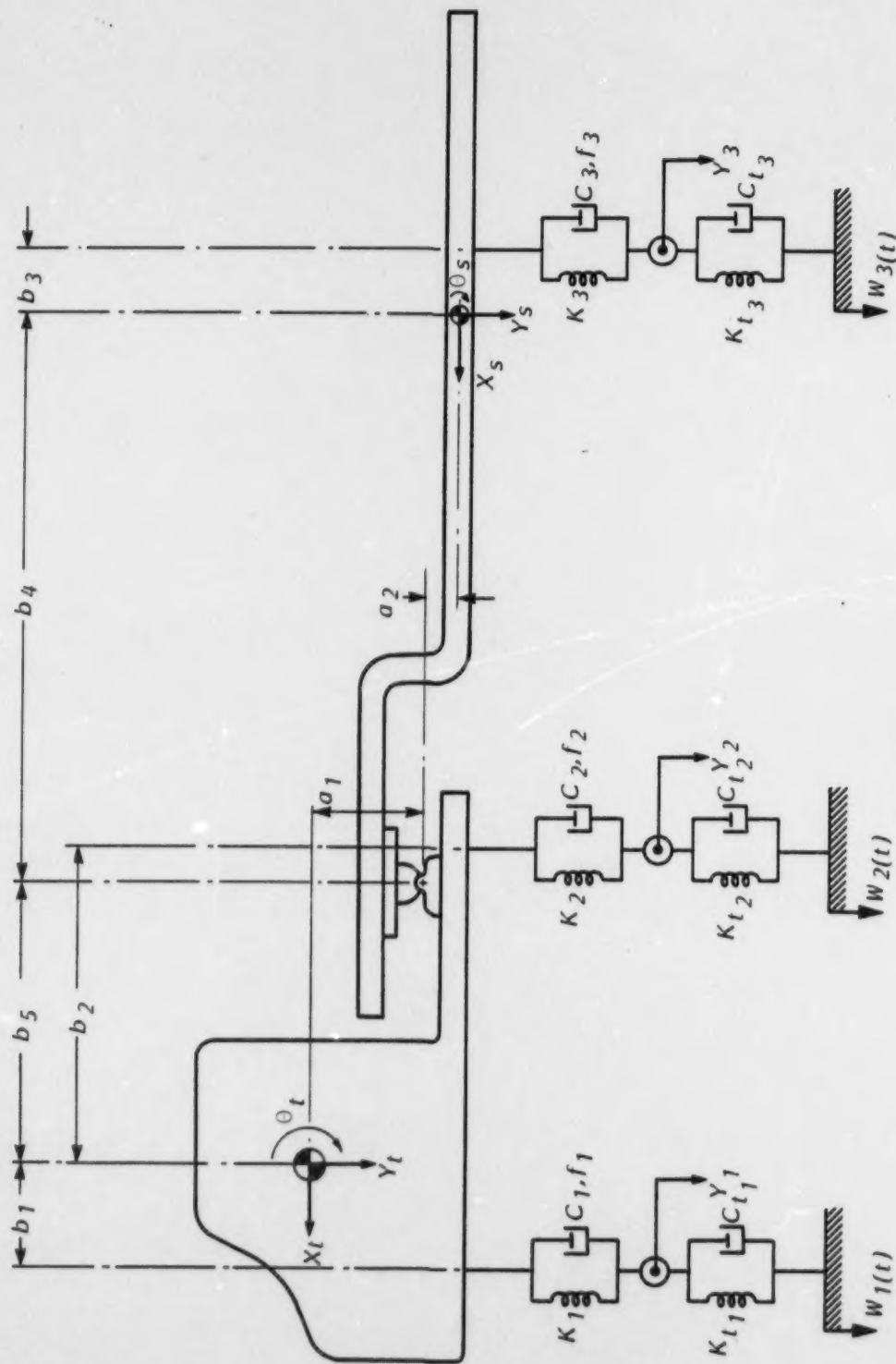


FIGURE 16  
THE TRACTOR - SEMITRAILER MODEL

.000051

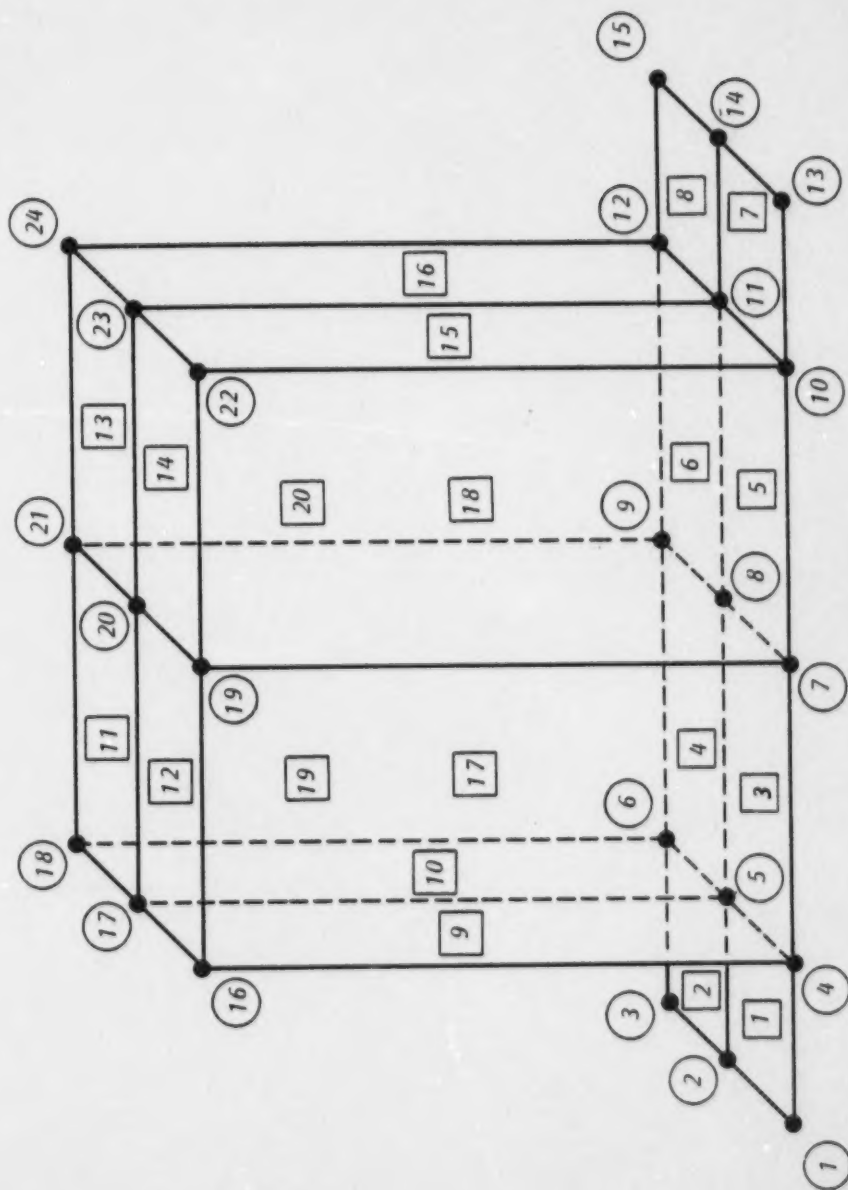


FIGURE 17  
FINITE ELEMENT MODEL FOR CASK/TIEDOWN/TRANSPORT FRAME SYSTEM

000052

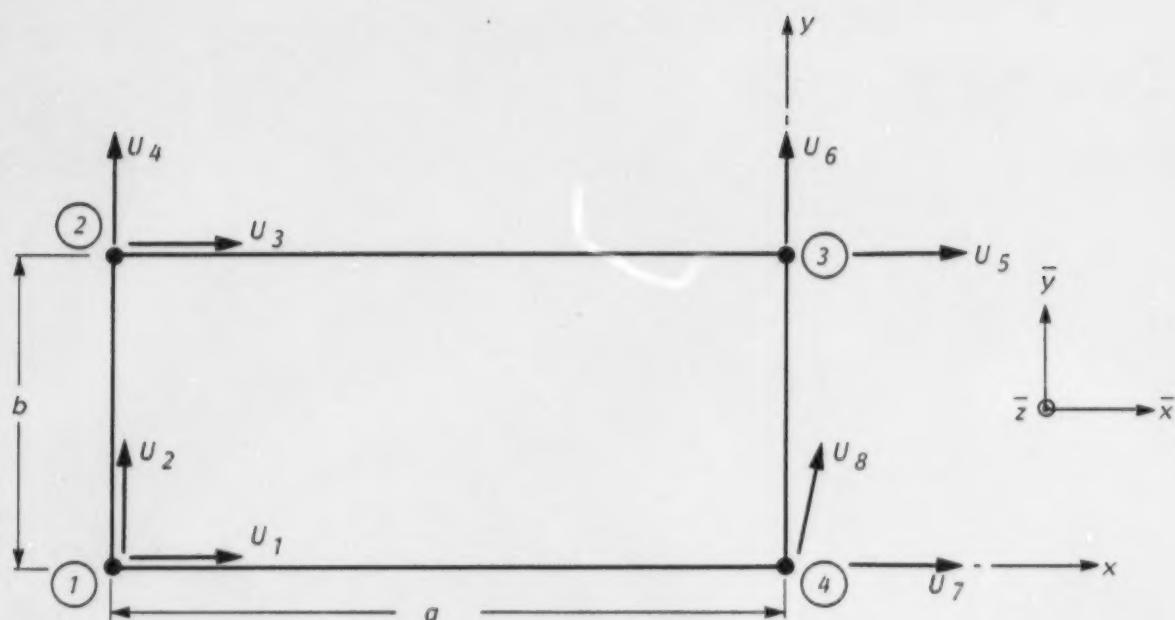


FIGURE 18  
RECTANGULAR PLATE ELEMENT (IN. PLANE FORCES)

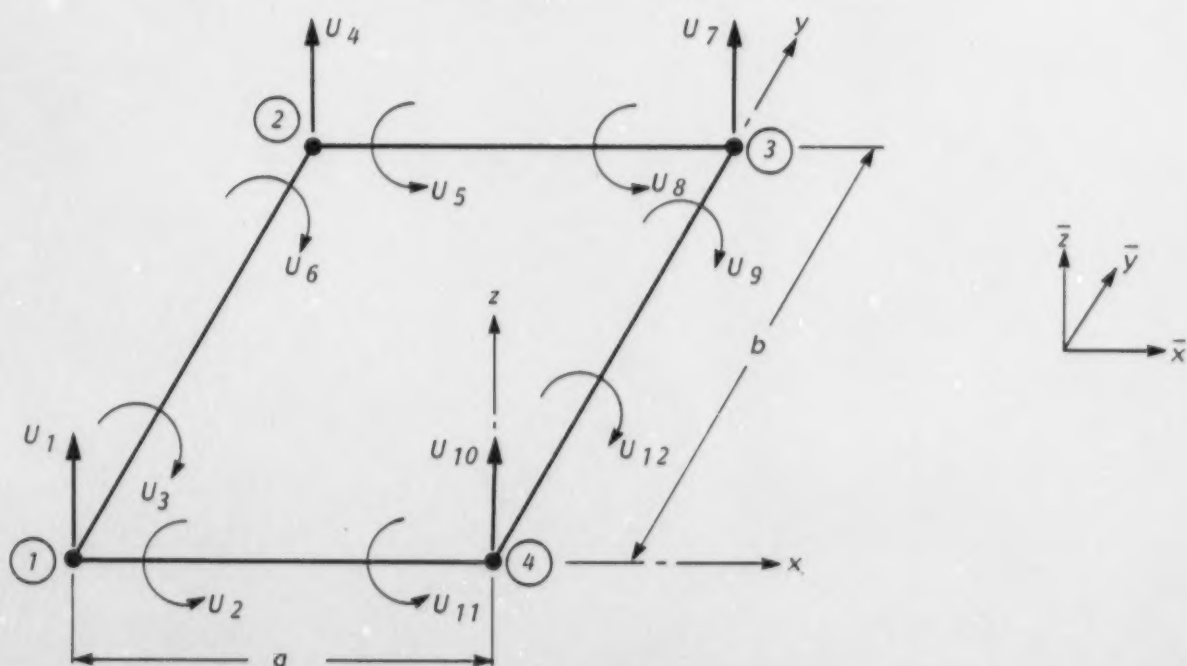


FIGURE 19  
RECTANGULAR PLATE ELEMENT (BENDING FORCES)

000053

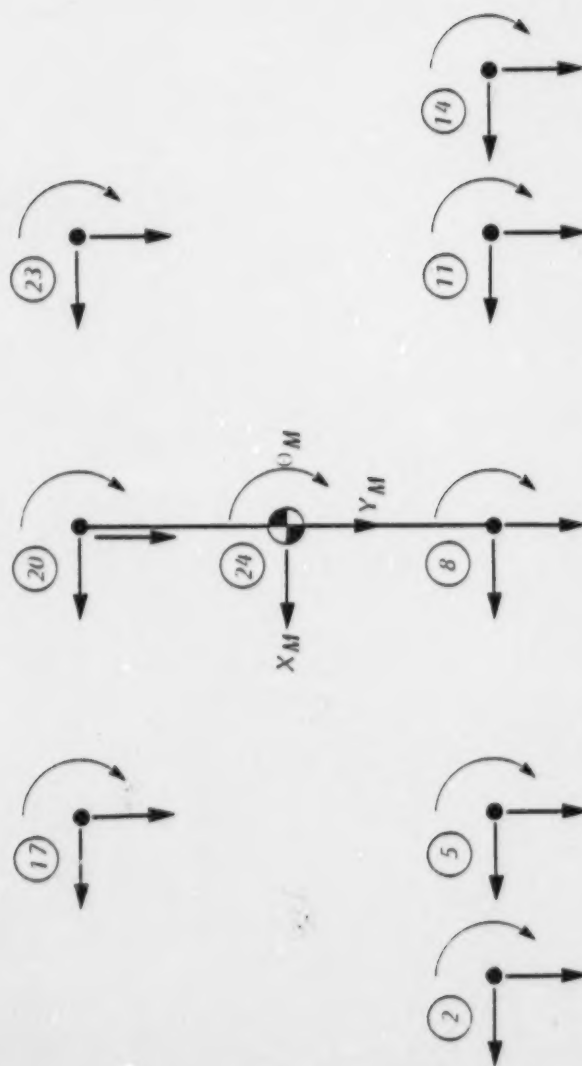


FIGURE 20  
REDUCED MODEL FOR CASK /TIEDOWN /TRANSPORT FRAME AND MODULE SYSTEM

000054

162385 RD

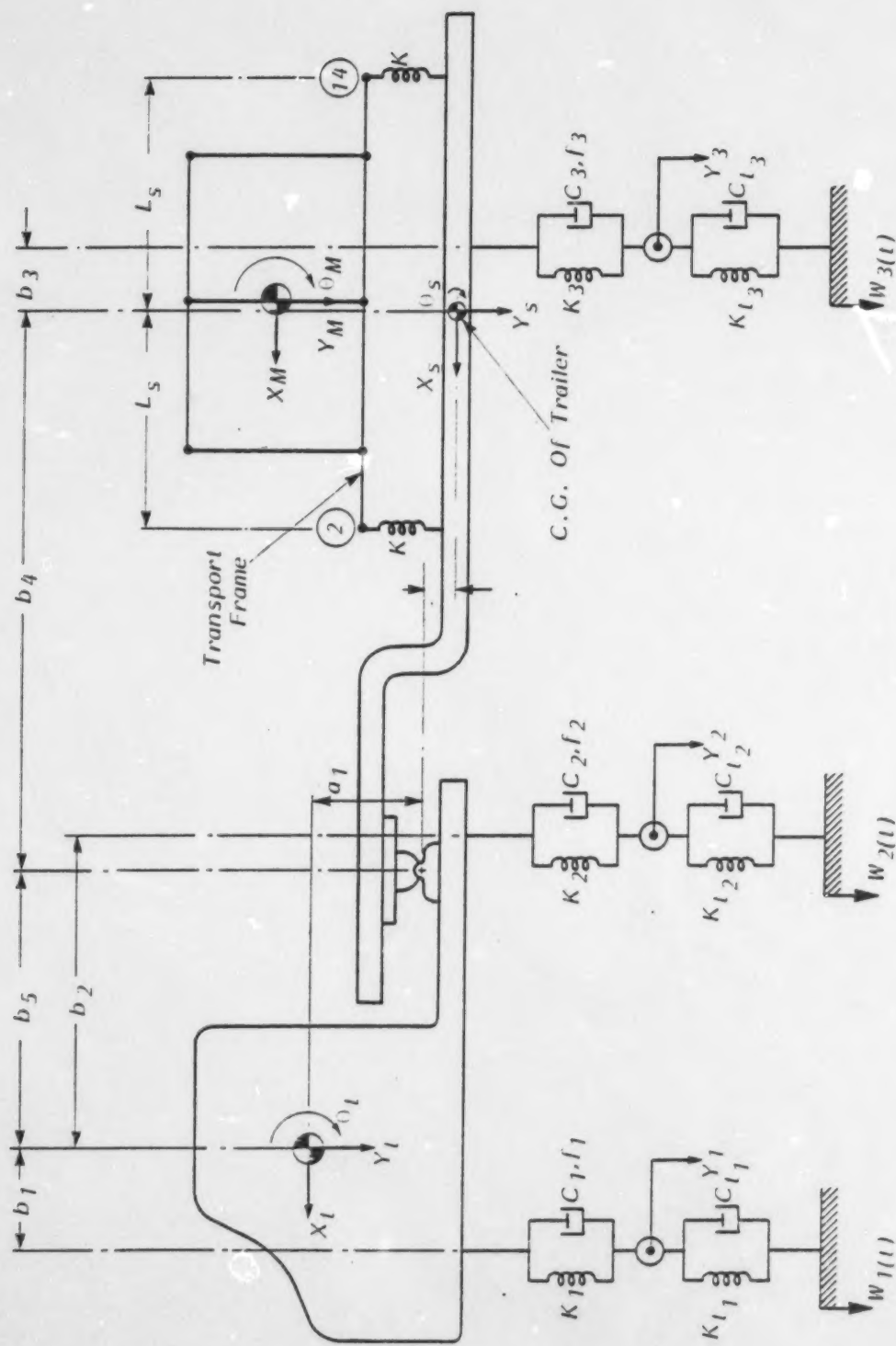


FIGURE 21  
ARTICULATED VEHICLE MODEL

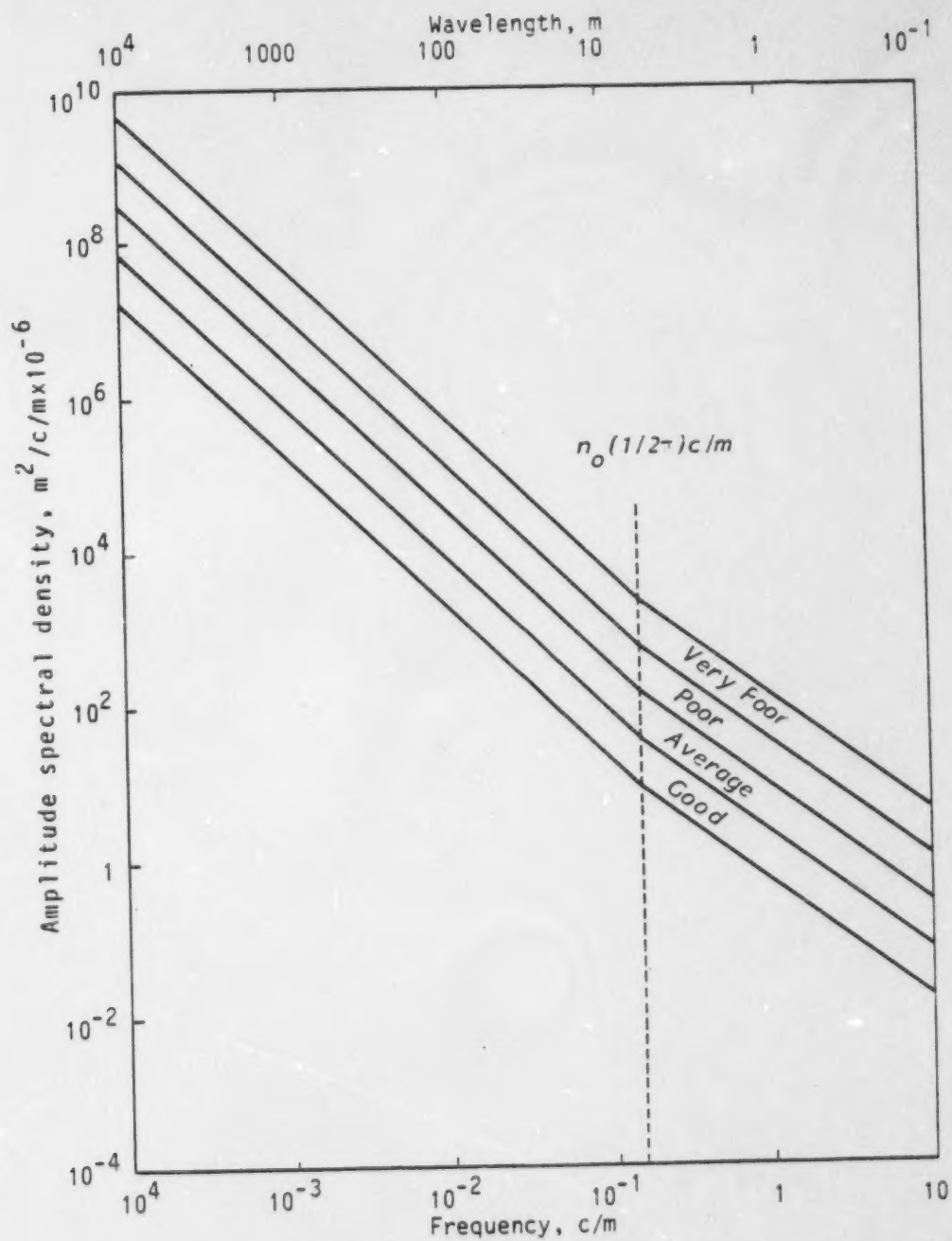


FIGURE 22  
CLASSIFICATION OF ROAD SURFACES BY POWER SPECTRAL DENSITY

000000

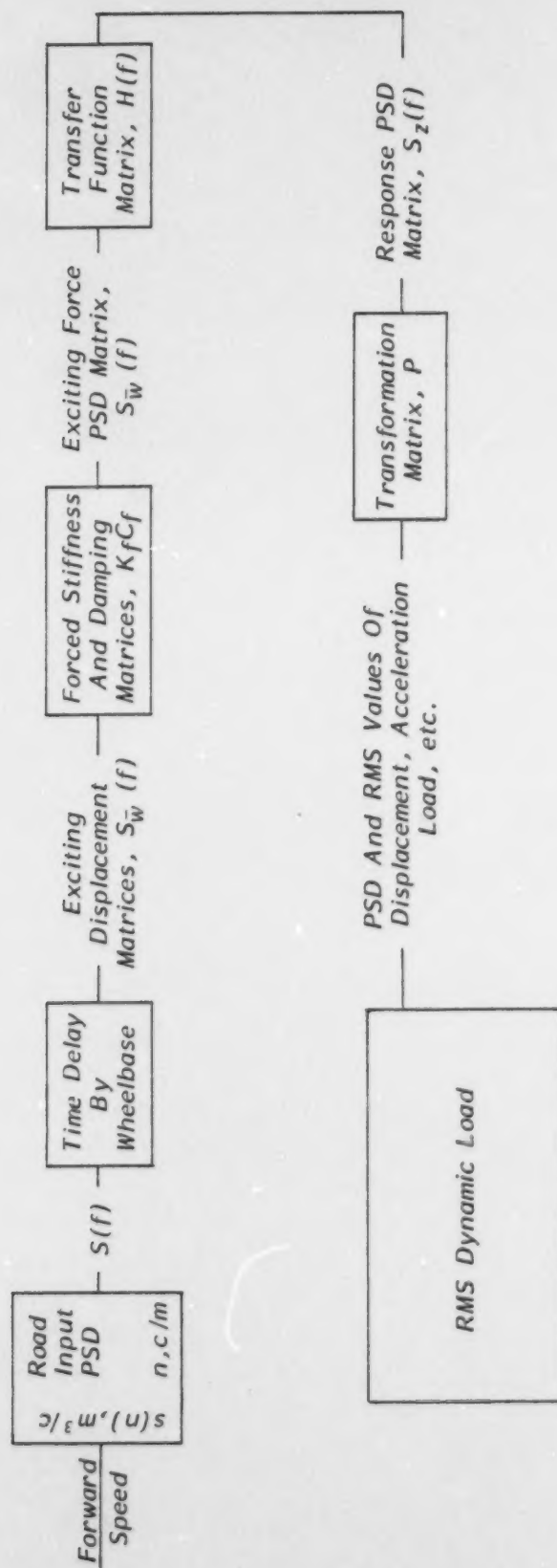


FIGURE 23  
VEHICLE RIDE ANALYSIS

000057

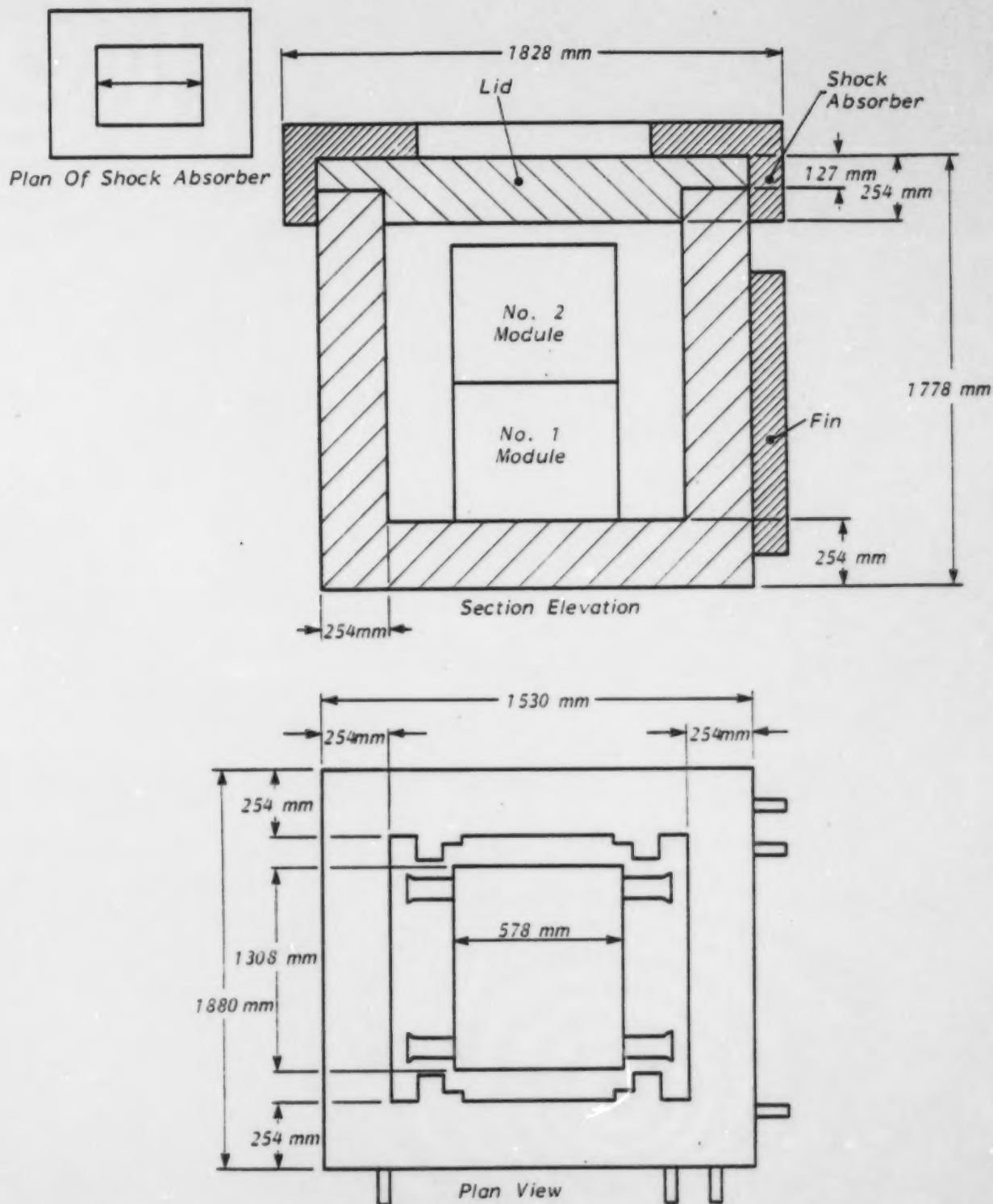


FIGURE 24  
ROAD CASK

000058

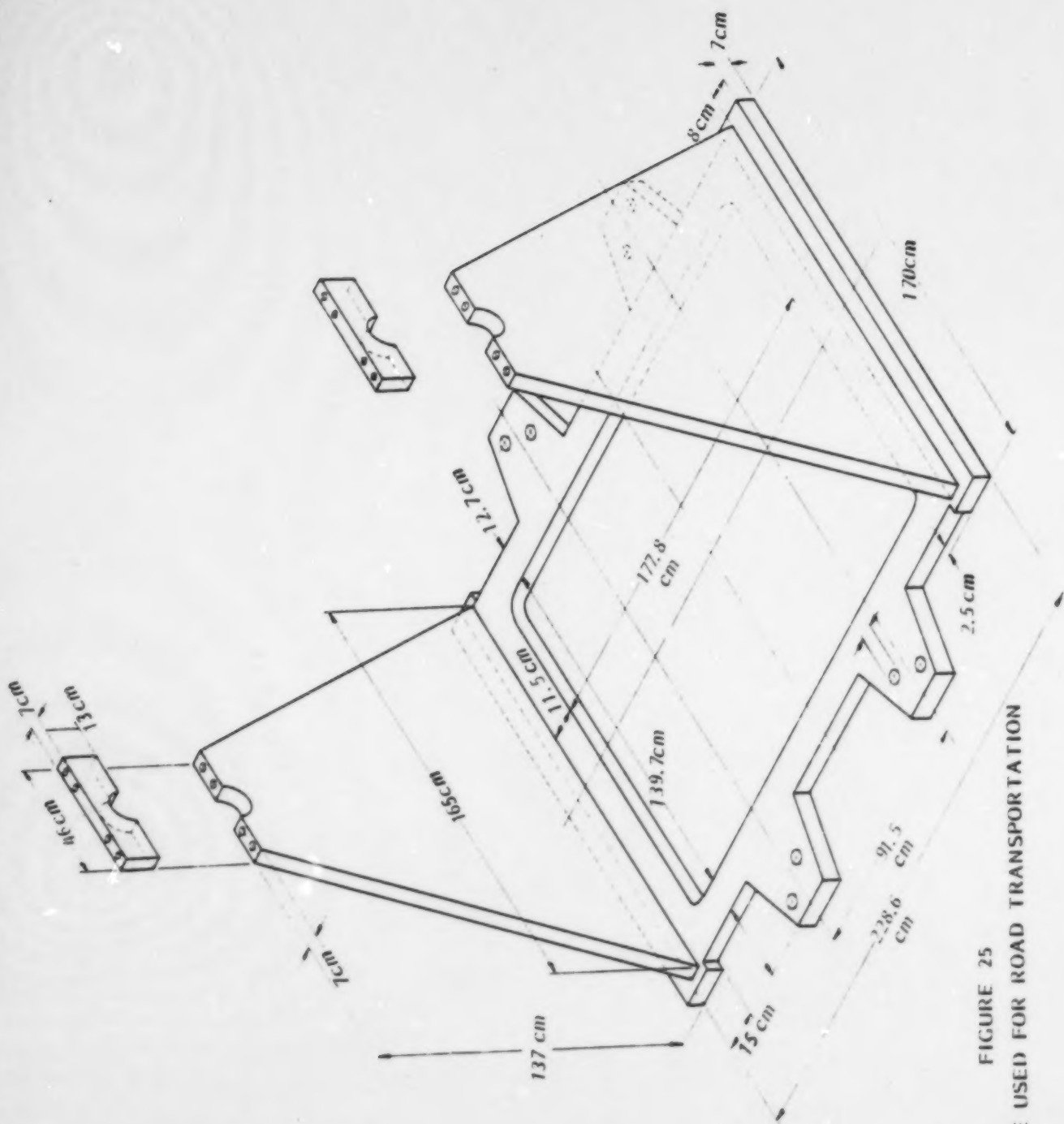


FIGURE 25  
SUPPORT FRAME USED FOR ROAD TRANSPORTATION

000059

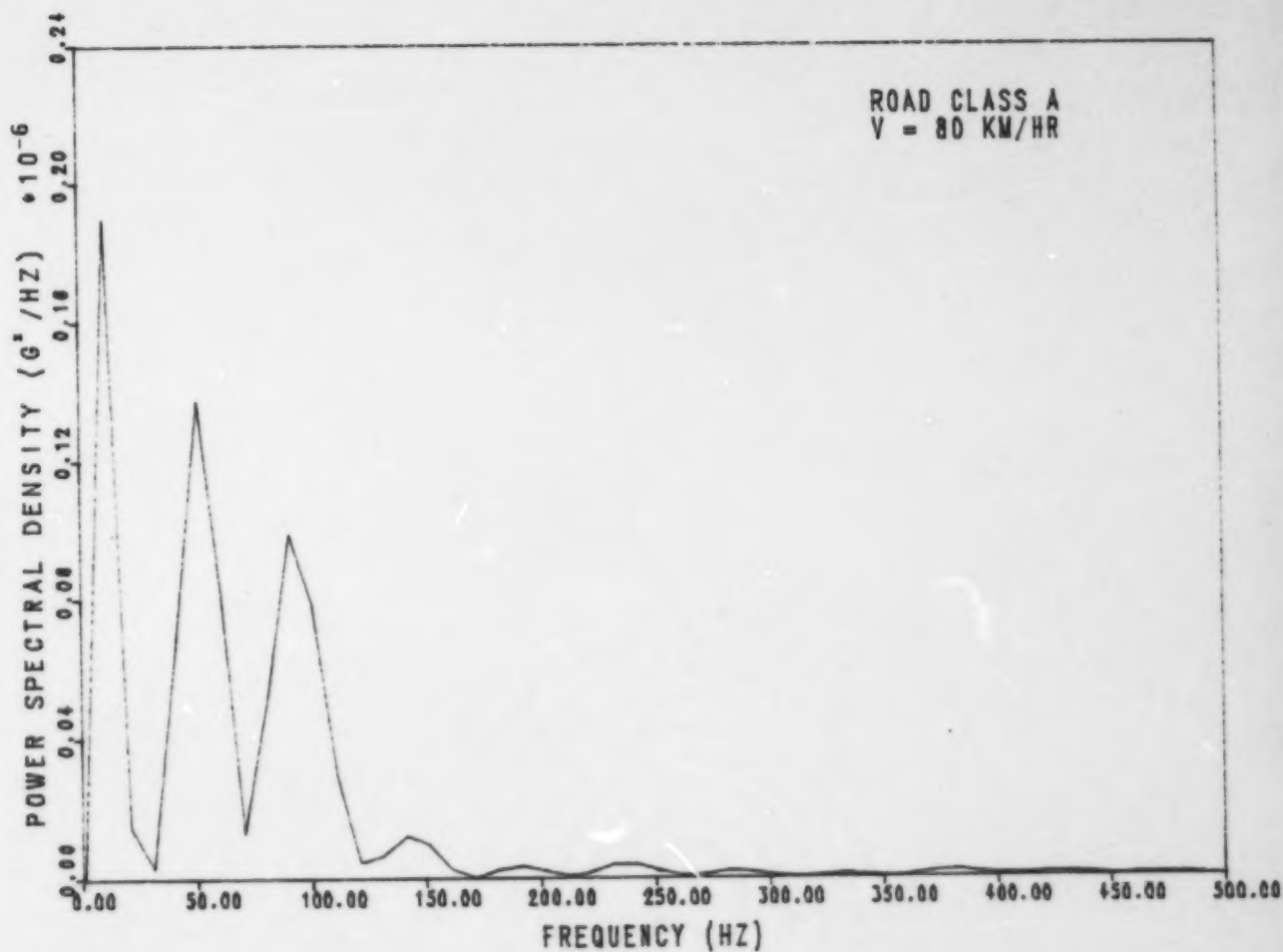


FIGURE 26  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL  
ACCELERATION AT THE TIE DOWN CONNECTION WITH THE  
TRAILER VERSUS FREQUENCY FOR ROAD CLASS A

000060

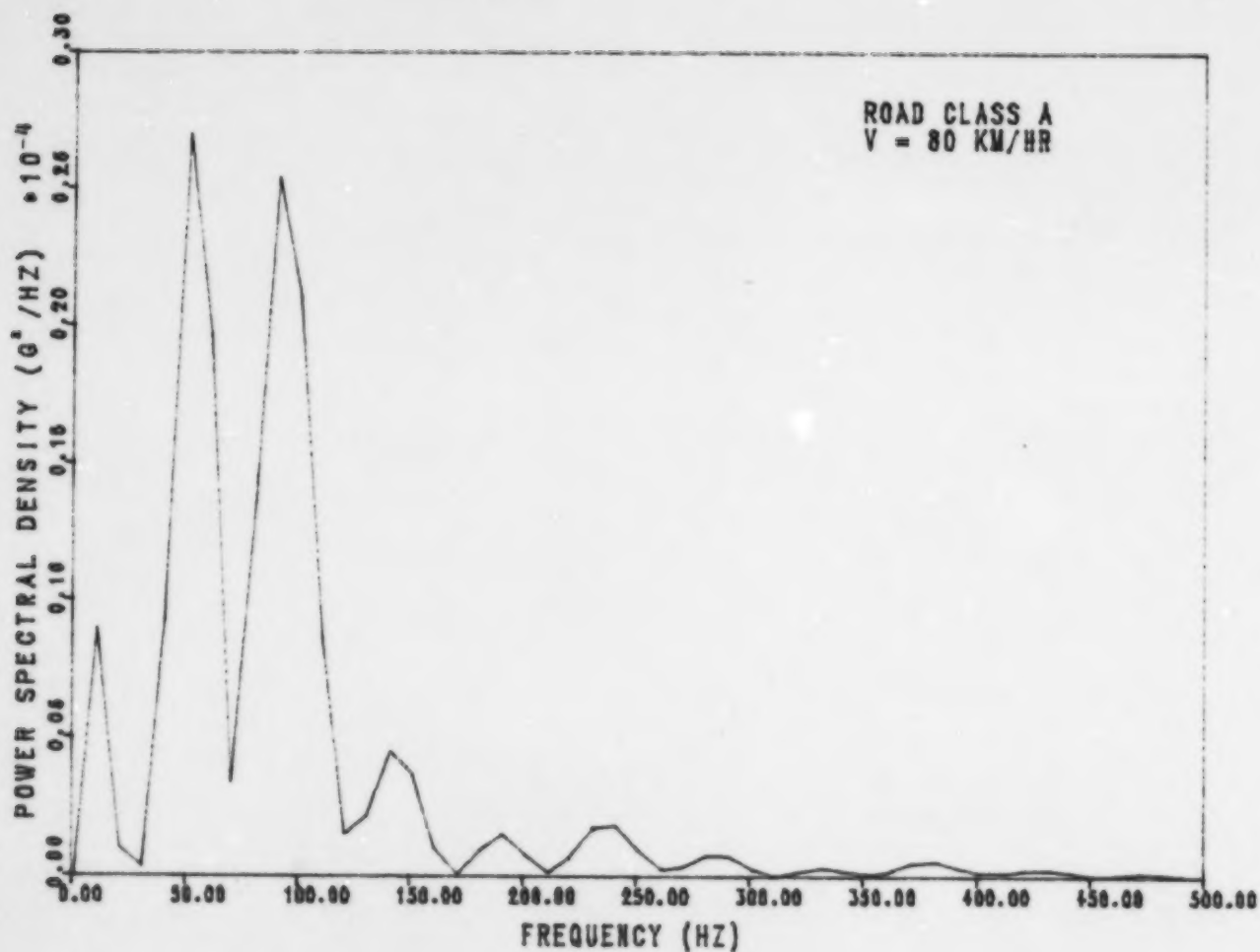


FIGURE 27

THE POWER SPECTRAL DENSITY OF THE VERTICAL ACCELERATION AT THE  
TIE DOWN CONNECTION WITH THE TRAILER VERSUS FREQUENCY FOR ROAD  
CLASS A

000061

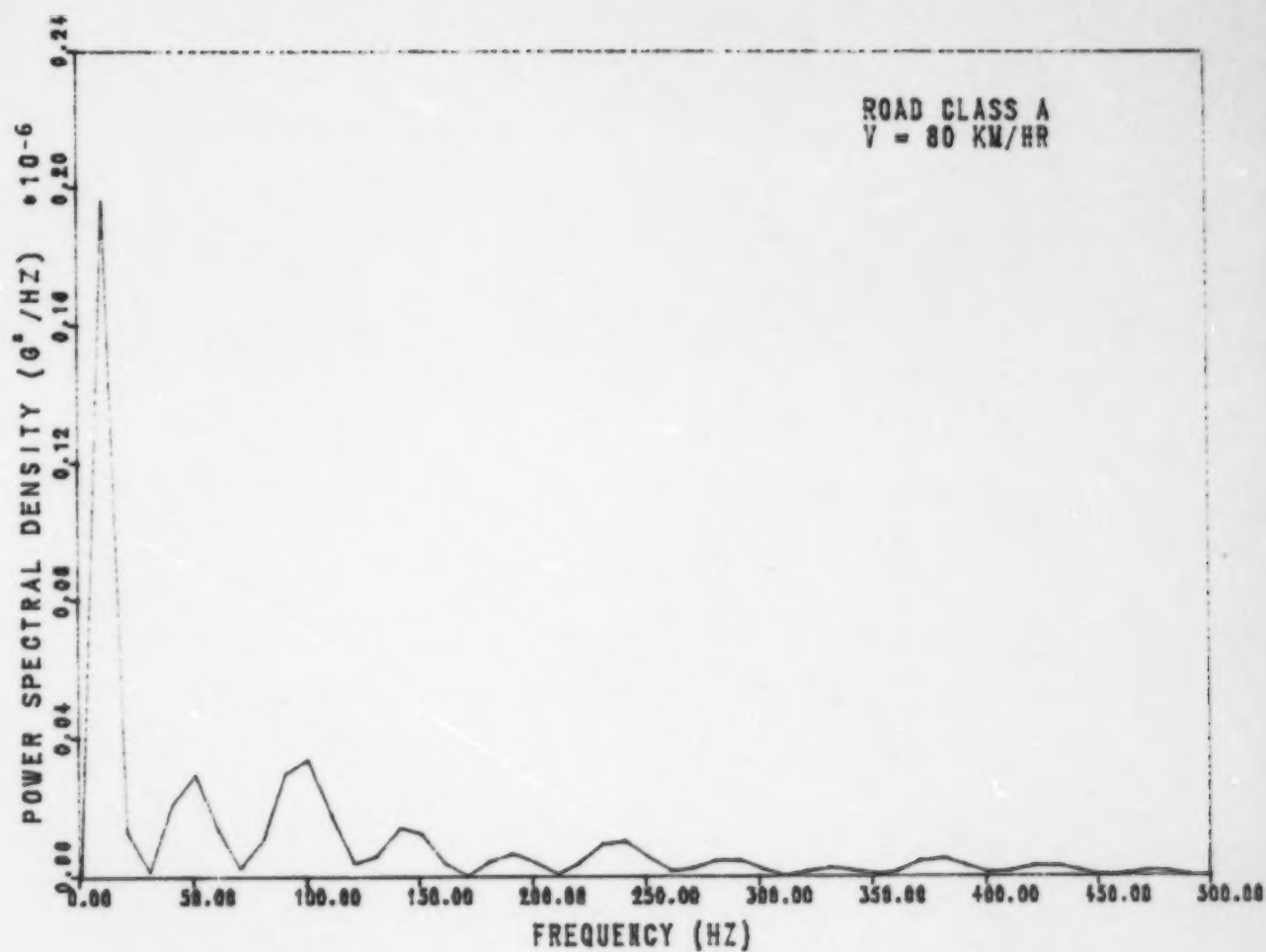


FIGURE 28

THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE CASK VERSUS  
FREQUENCY FOR ROAD CLASS A

000062

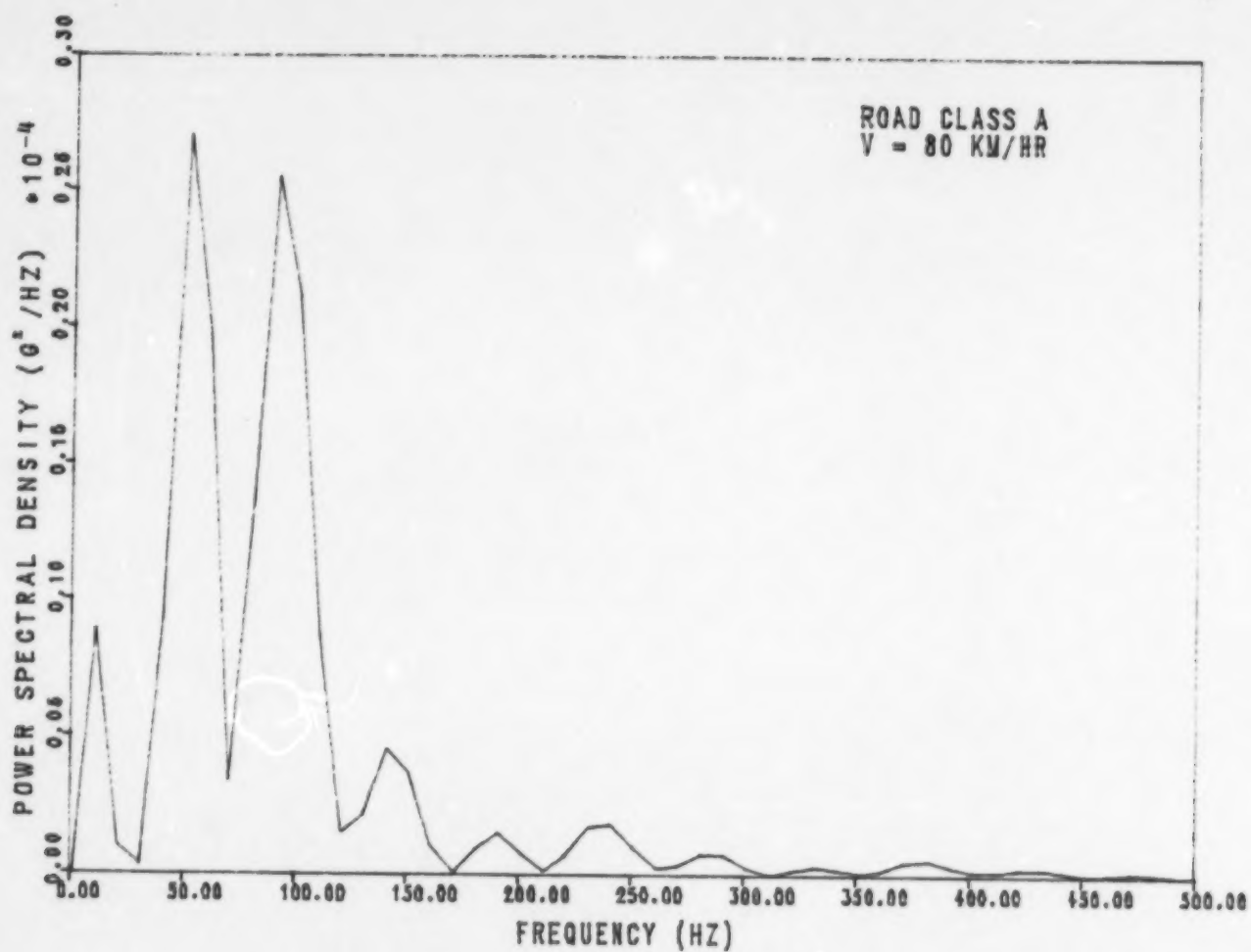


FIGURE 29

THE POWER SPECTRAL DENSITY OF THE VERTICAL ACCELERATION AT THE TIE DOWN CONNECTION WITH THE CASK VERSUS FREQUENCY FOR ROAD CLASS A

000063

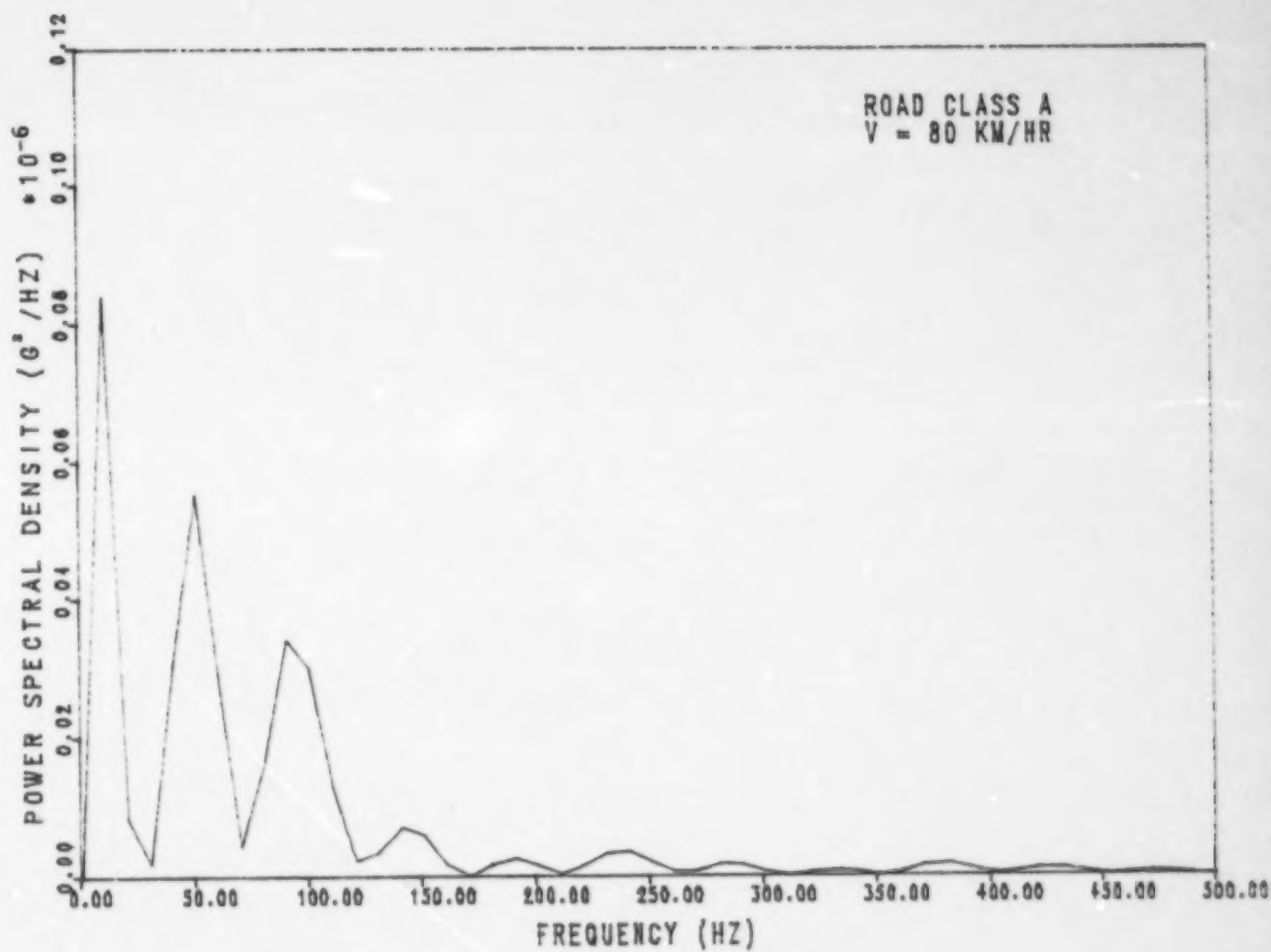


FIGURE 30  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
OF THE MODULE STACK CENTRE OF GRAVITY VERSUS FREQUENCY  
FOR ROAD CLASS A

000064

162395 RD

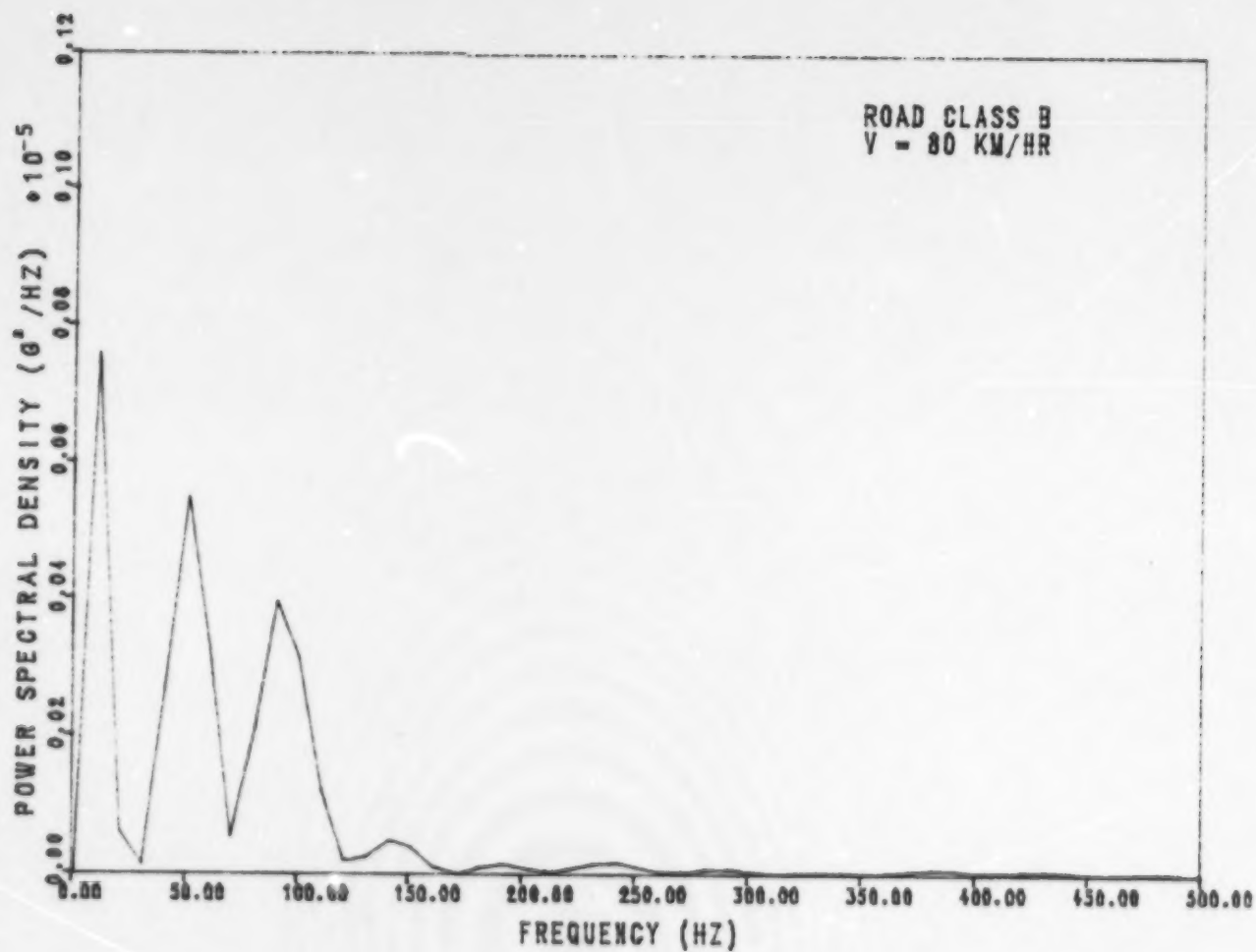


FIGURE 31  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE TRAILER VERSUS  
FREQUENCY FOR ROAD CLASS B

000065

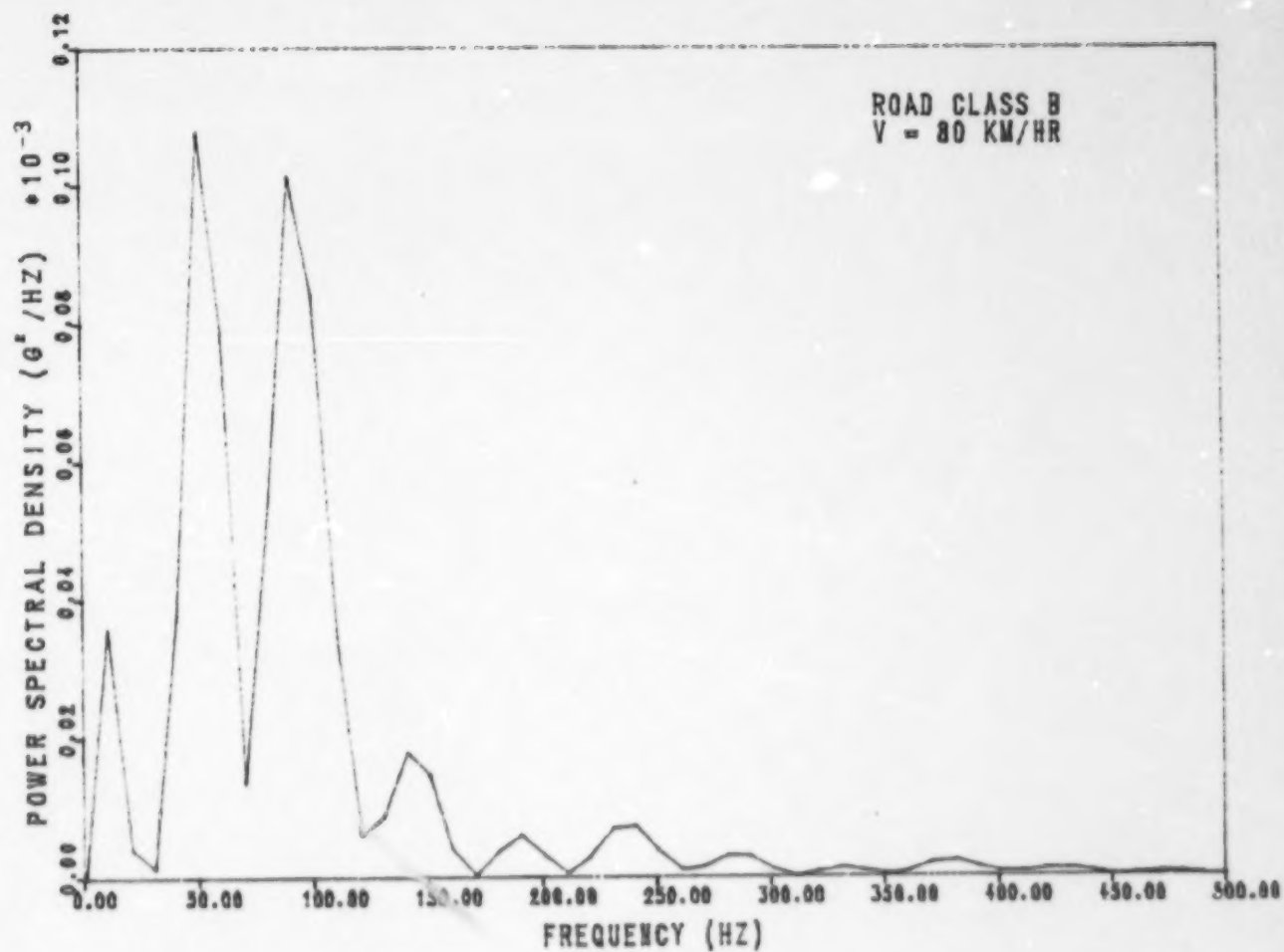


FIGURE 32  
THE POWER SPECTRAL DENSITY OF THE VERTICAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE TRAILER VERSUS  
FREQUENCY FOR ROAD CLASS B

000066

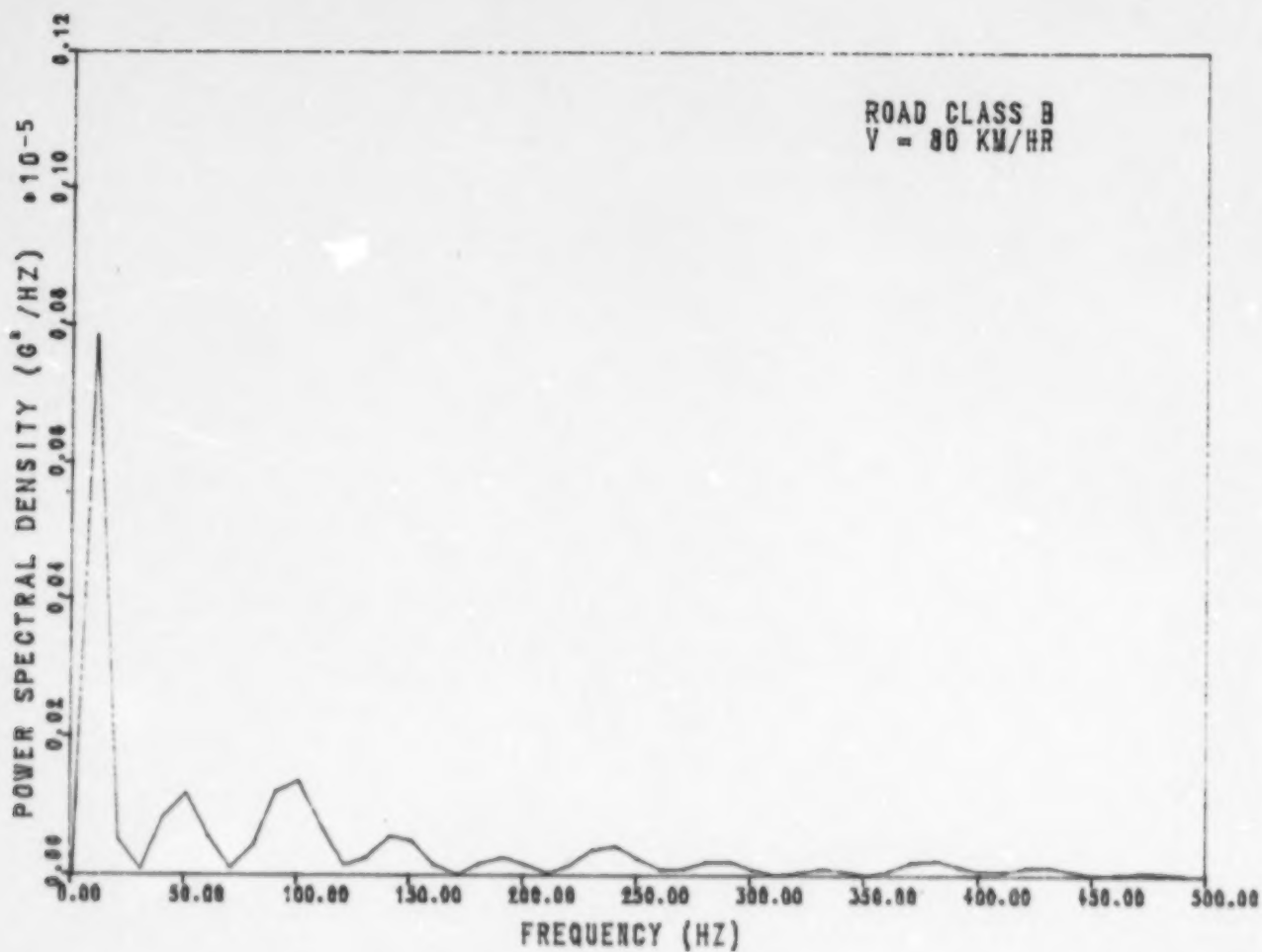


FIGURE 33  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE CASK VERSUS  
FREQUENCY FOR ROAD CLASS B

000067

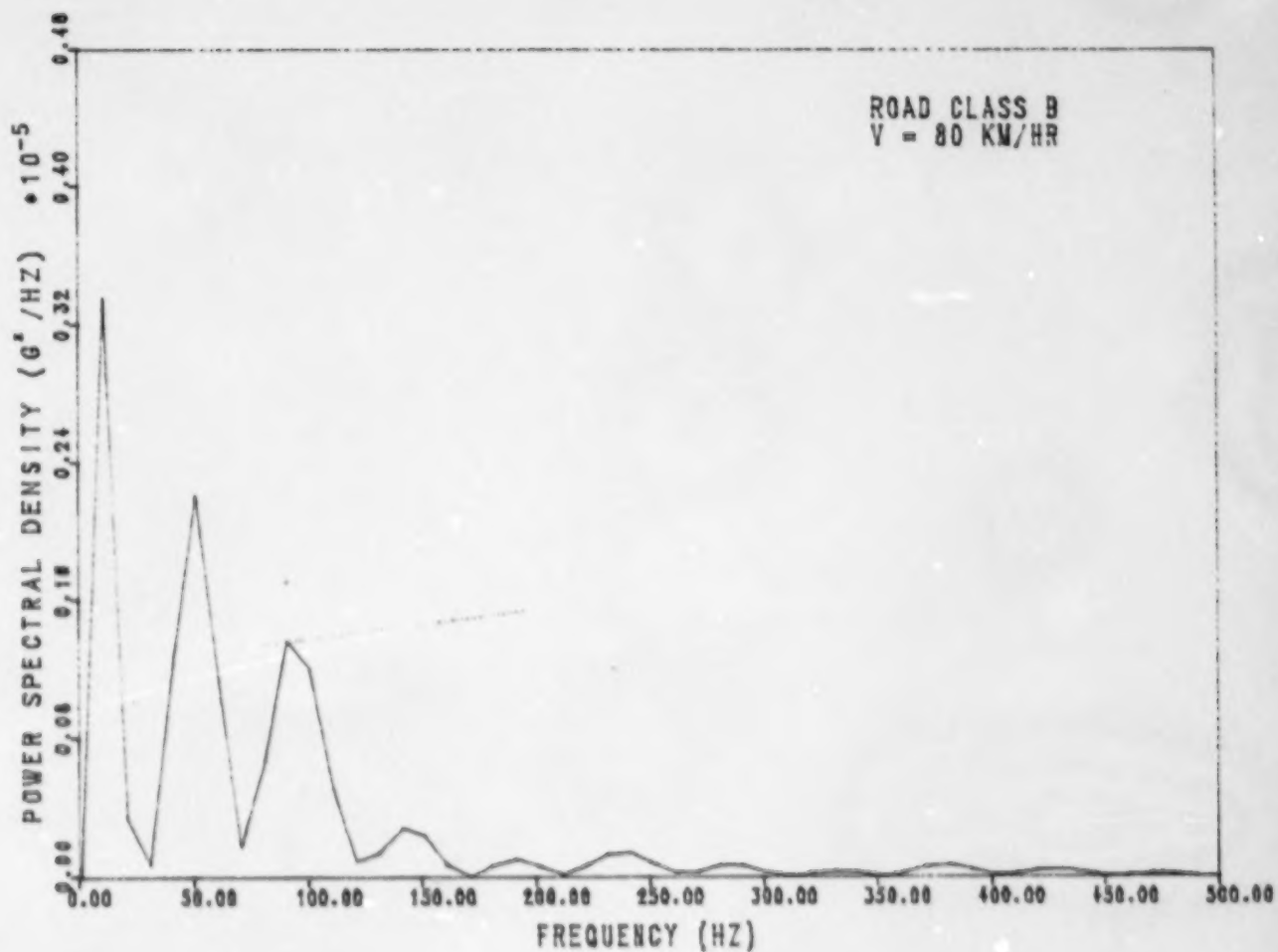


FIGURE 34  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
OF THE MODULE STACK CENTRE OF GRAVITY VERSUS FREQUENCY  
FOR ROAD CLASS B

000068

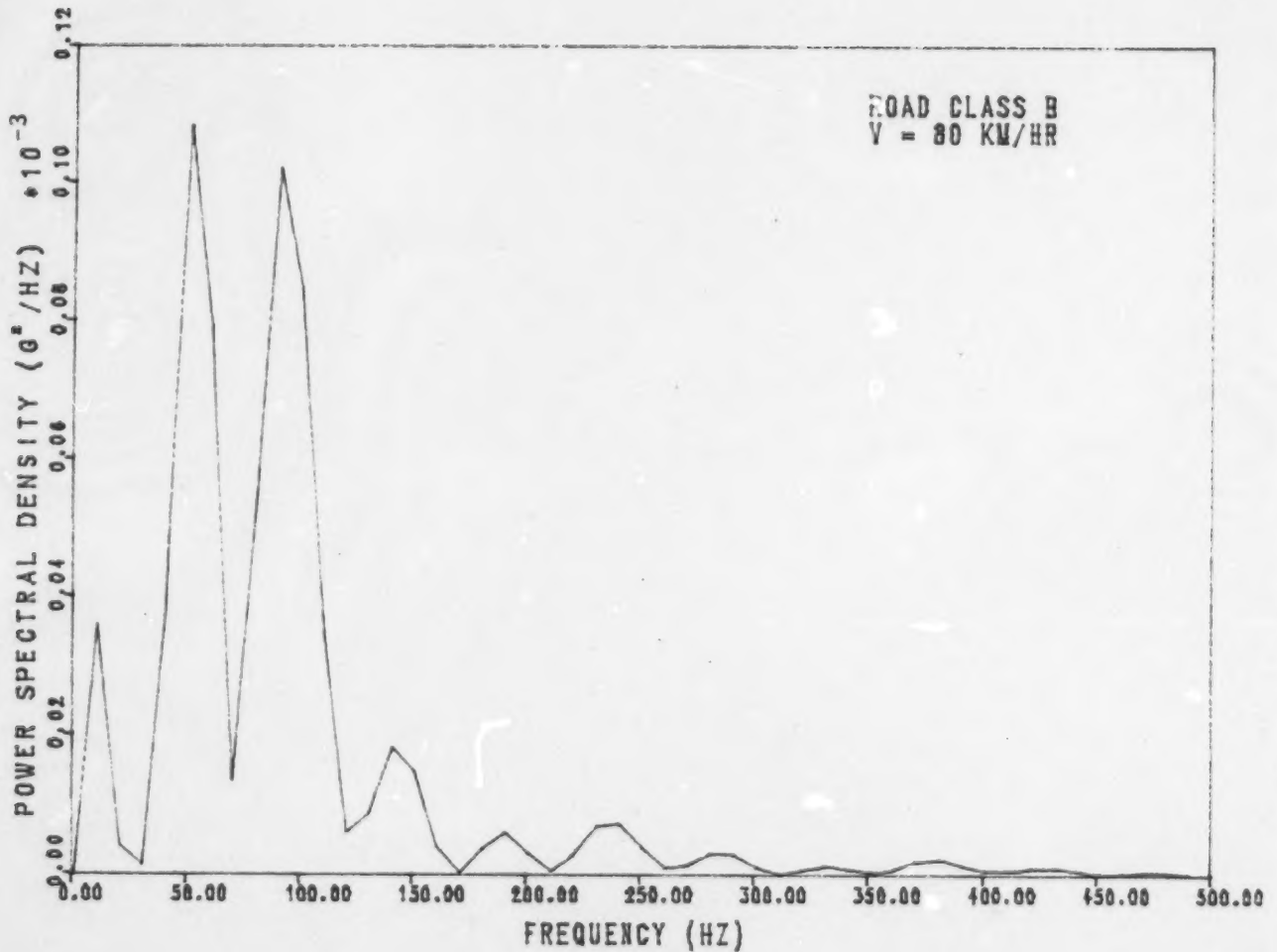


FIGURE 35  
THE POWER SPECTRAL DENSITY OF THE VERTICAL ACCELERATION  
OF THE MODULE STACK CENTRE OF GRAVITY VERSUS  
FREQUENCY FOR ROAD CLASS B

000069

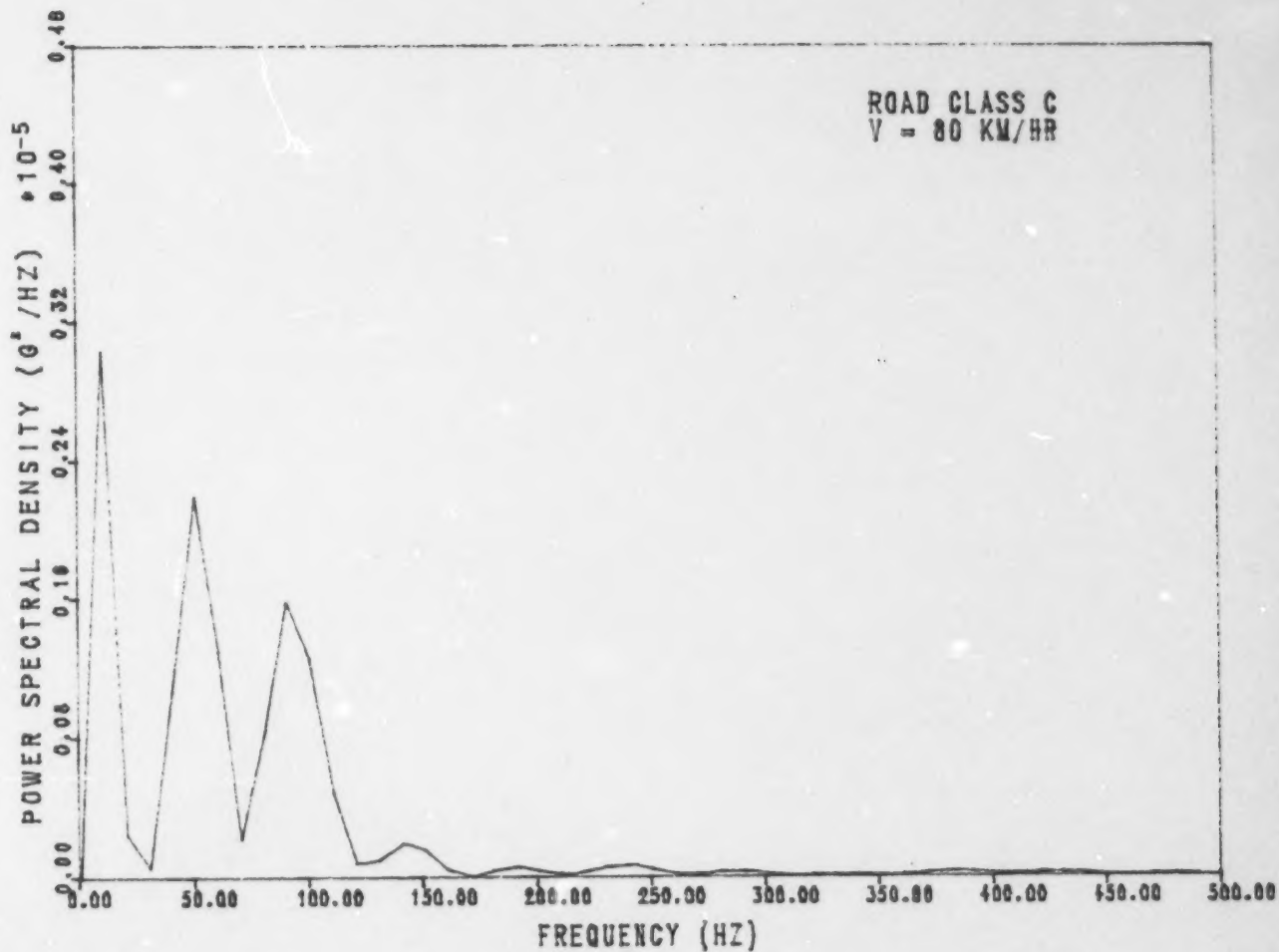


FIGURE 36  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE TRAILER VERSUS  
FREQUENCY FOR ROAD CLASS C

000070

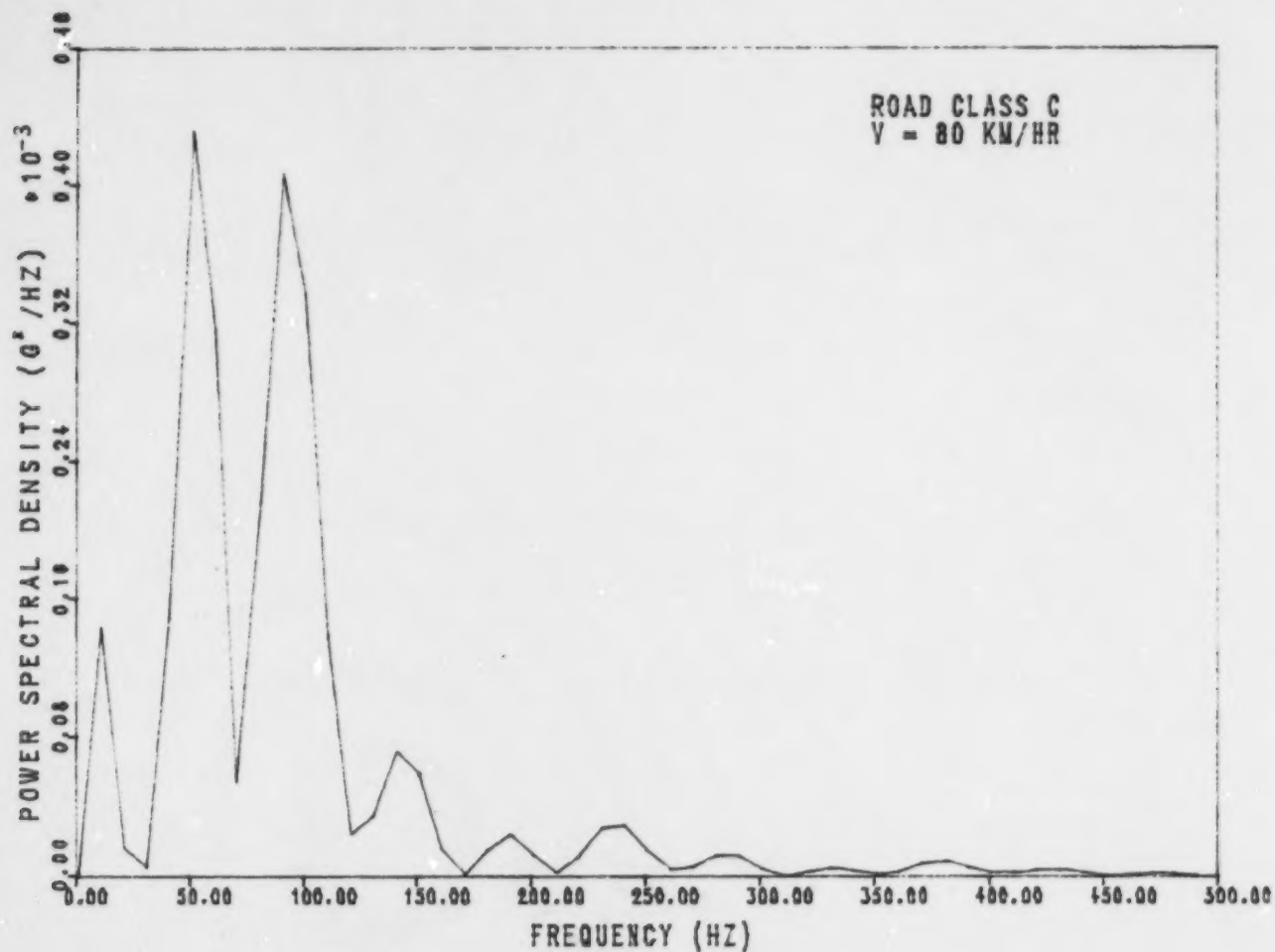


FIGURE 37

THE POWER SPECTRAL DENSITY OF THE VERTICAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE TRAILER VERSUS  
FREQUENCY FOR ROAD CLASS C

000071

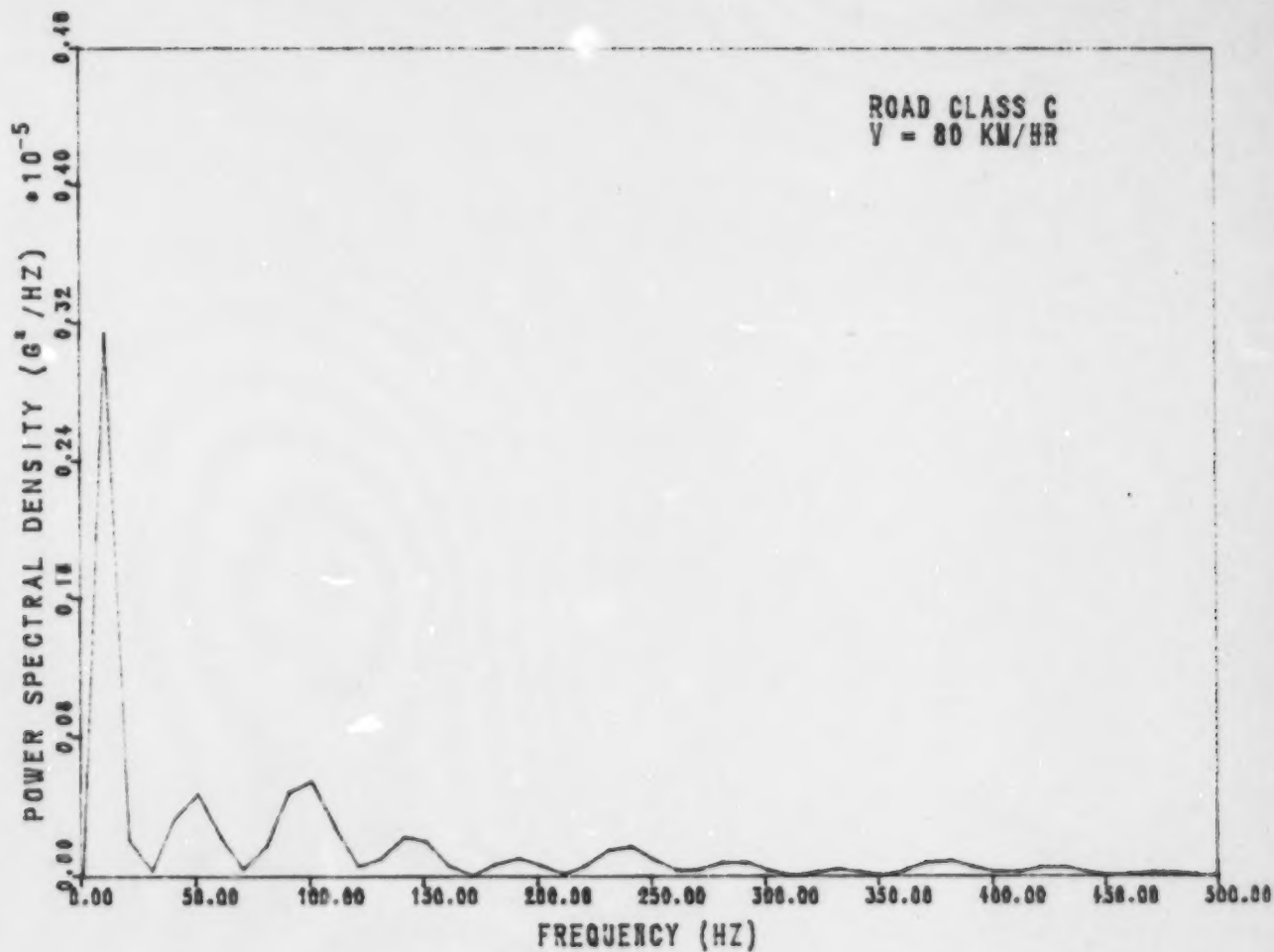


FIGURE 38  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE CASK VERSUS  
FREQUENCY FOR ROAD CLASS C

000072

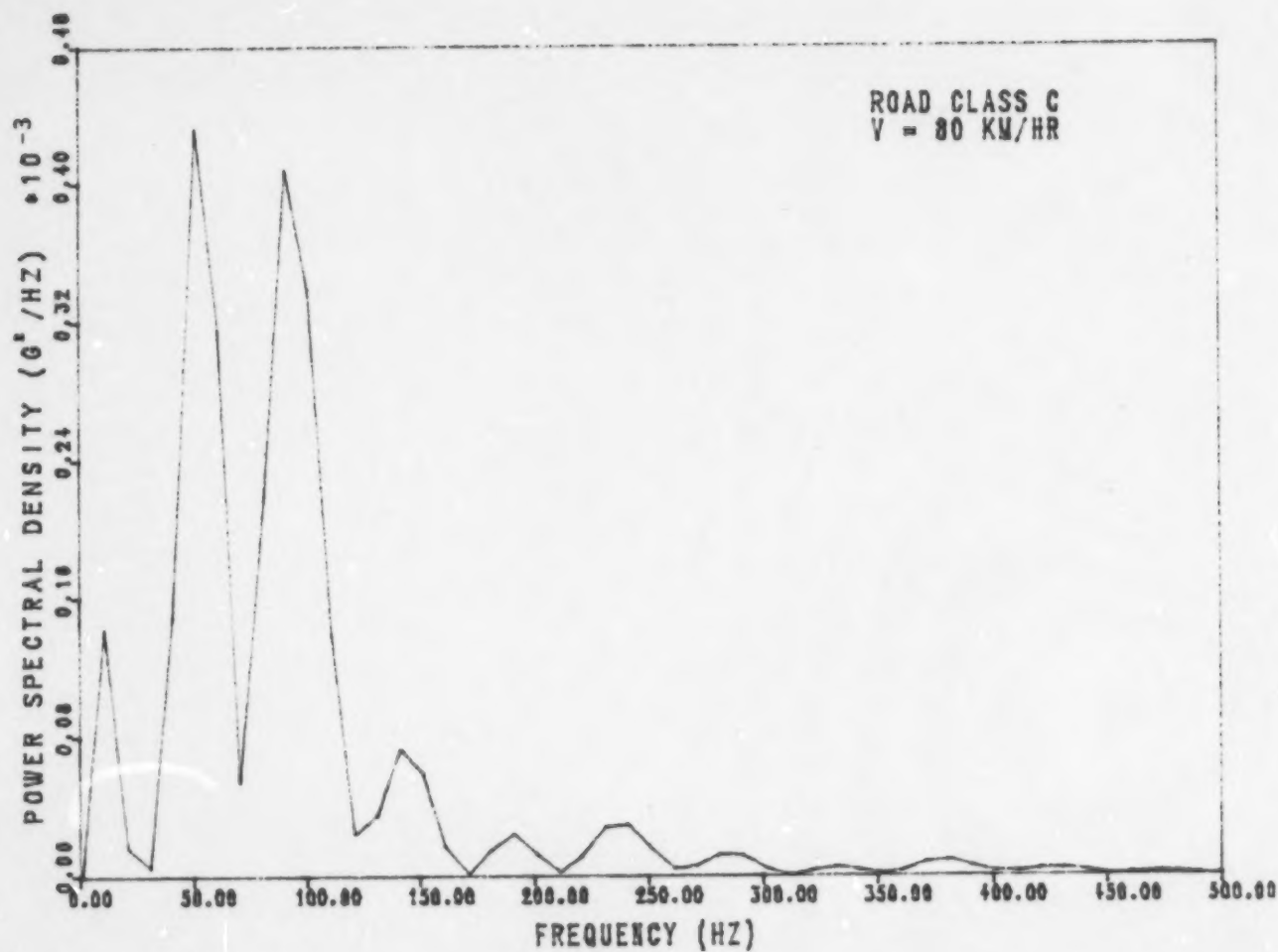


FIGURE 39

THE POWER SPECTRAL DENSITY OF THE VERTICAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE CASK VERSUS  
FREQUENCY FOR ROAD CLASS C

000073

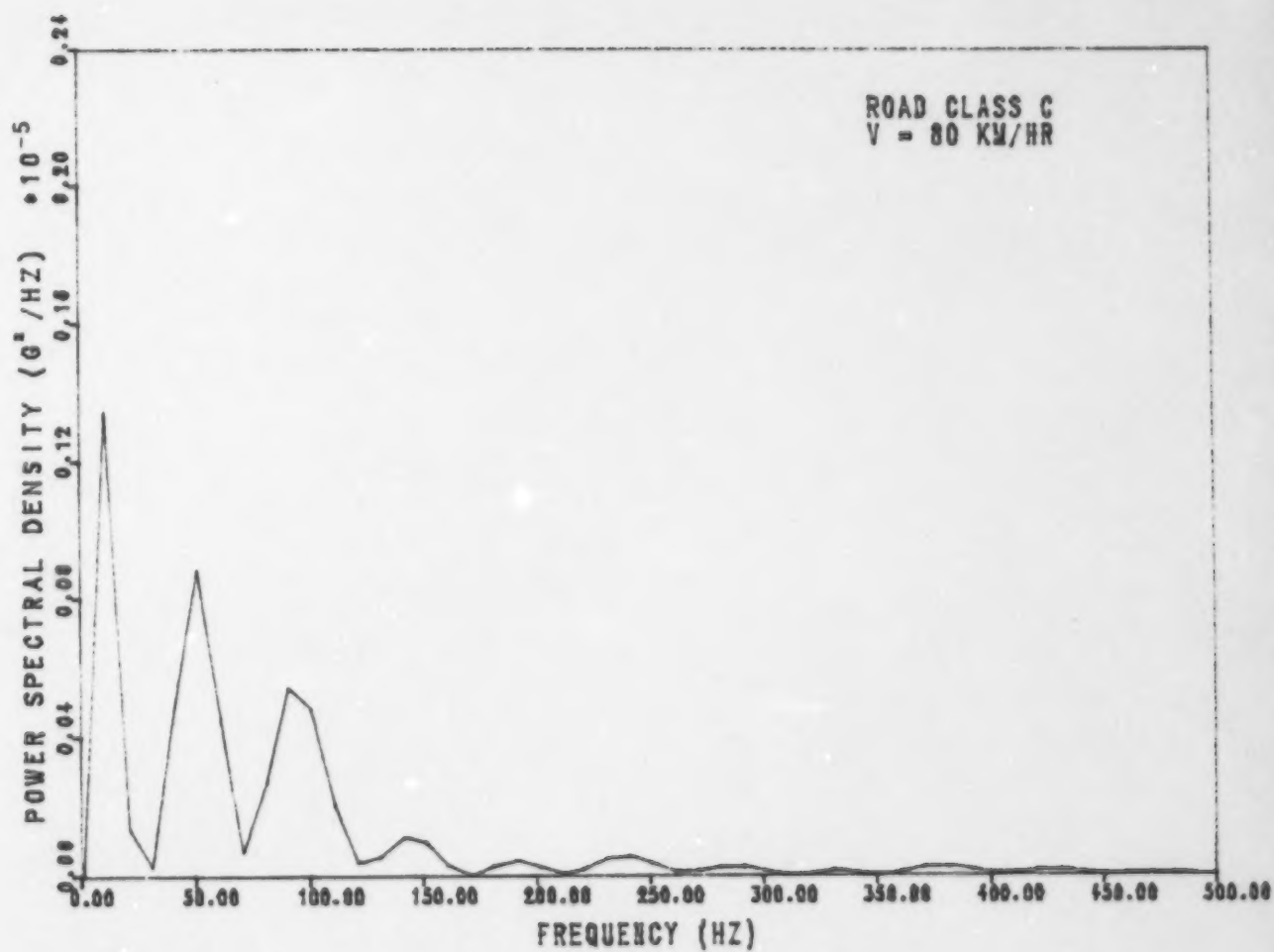


FIGURE 40  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
OF THE MODULE STACK CENTRE OF GRAVITY VERSUS  
FREQUENCY FOR ROAD CLASS C

000074

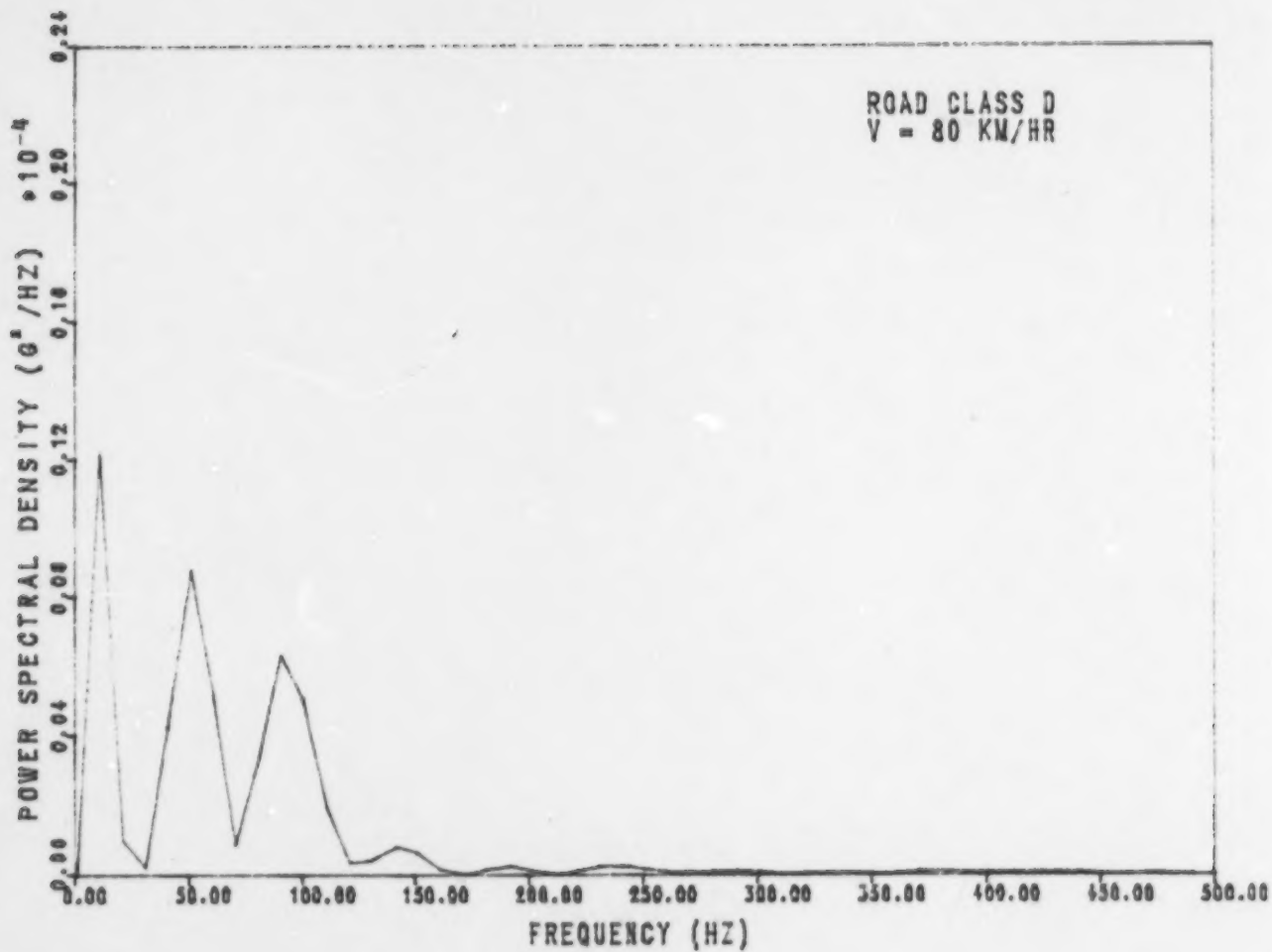


FIGURE 41  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE TRAILER VERSUS  
FREQUENCY FOR ROAD CLASS D

00007

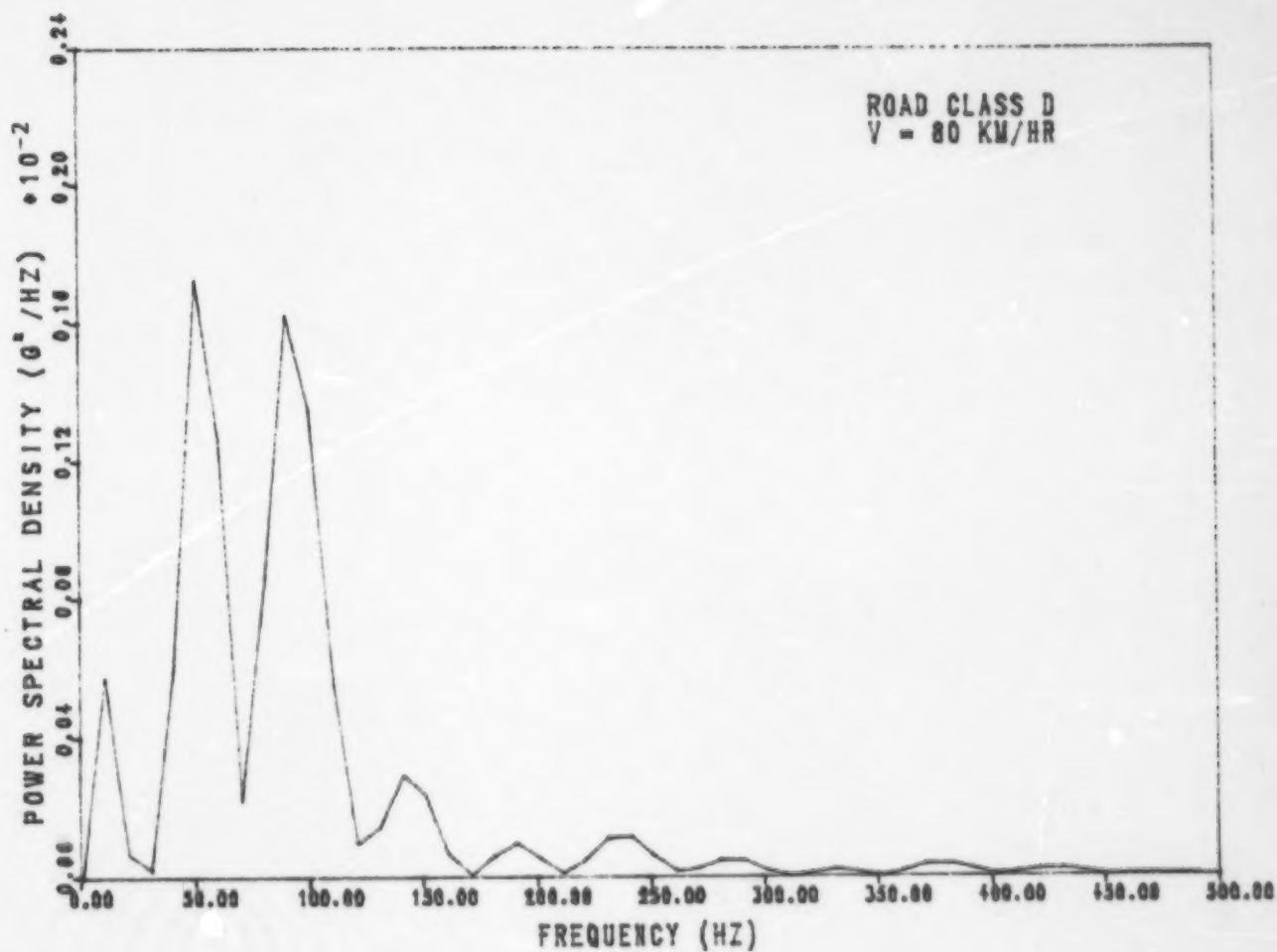


FIGURE 42  
THE POWER SPECTRAL DENSITY OF THE VERTICAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE TRAILER VERSUS  
FREQUENCY FOR ROAD CLASS D

000076

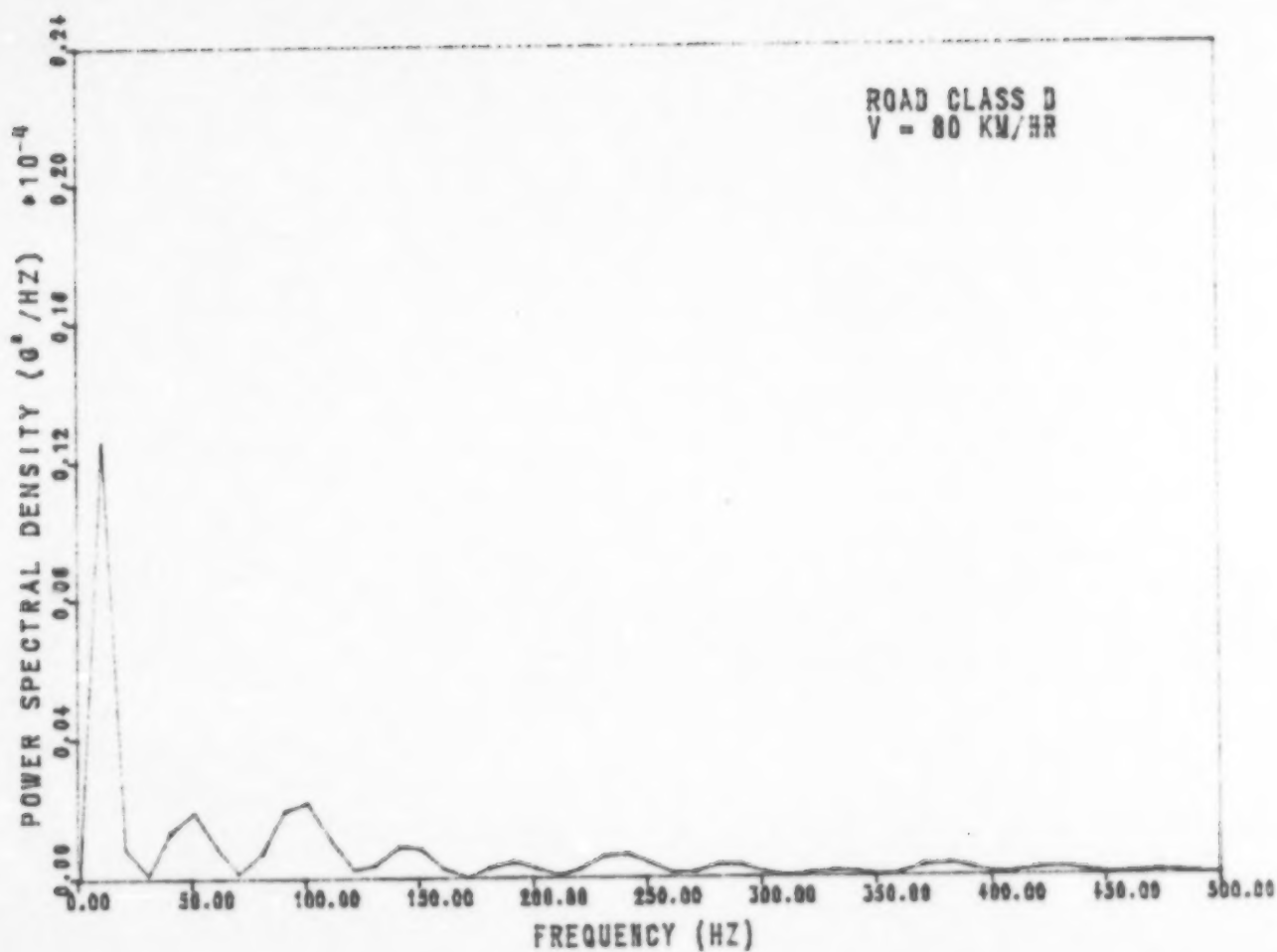


FIGURE 43

THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
AT THE TIE DOWN CONNECTION WITH THE CASK VERSUS  
FREQUENCY FOR ROAD CLASS D

000077

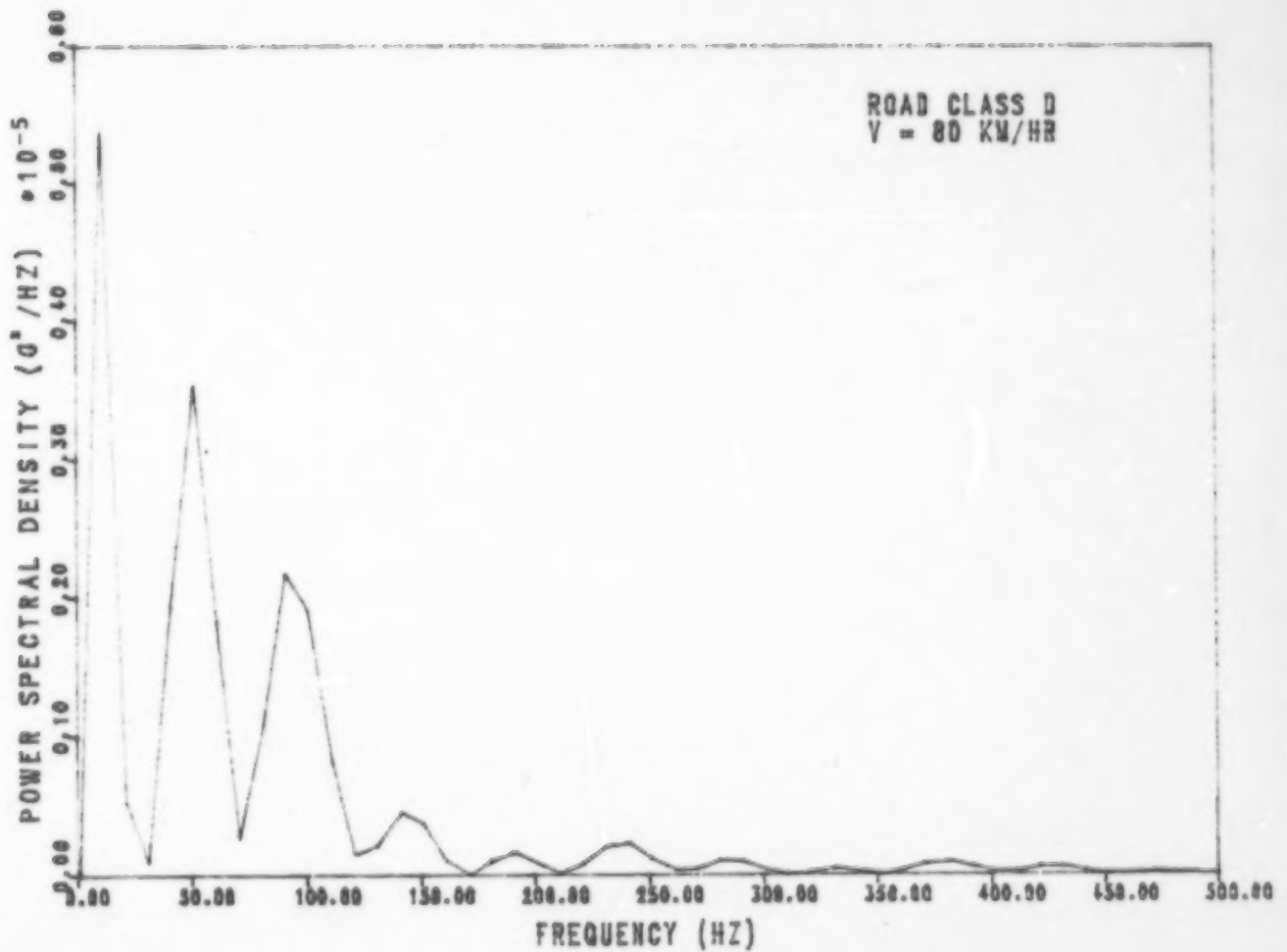


FIGURE 44  
THE POWER SPECTRAL DENSITY OF THE LONGITUDINAL ACCELERATION  
OF THE MODULE STACK CENTRE OF GRAVITY VERSUS  
FREQUENCY FOR ROAD CLASS D

000078

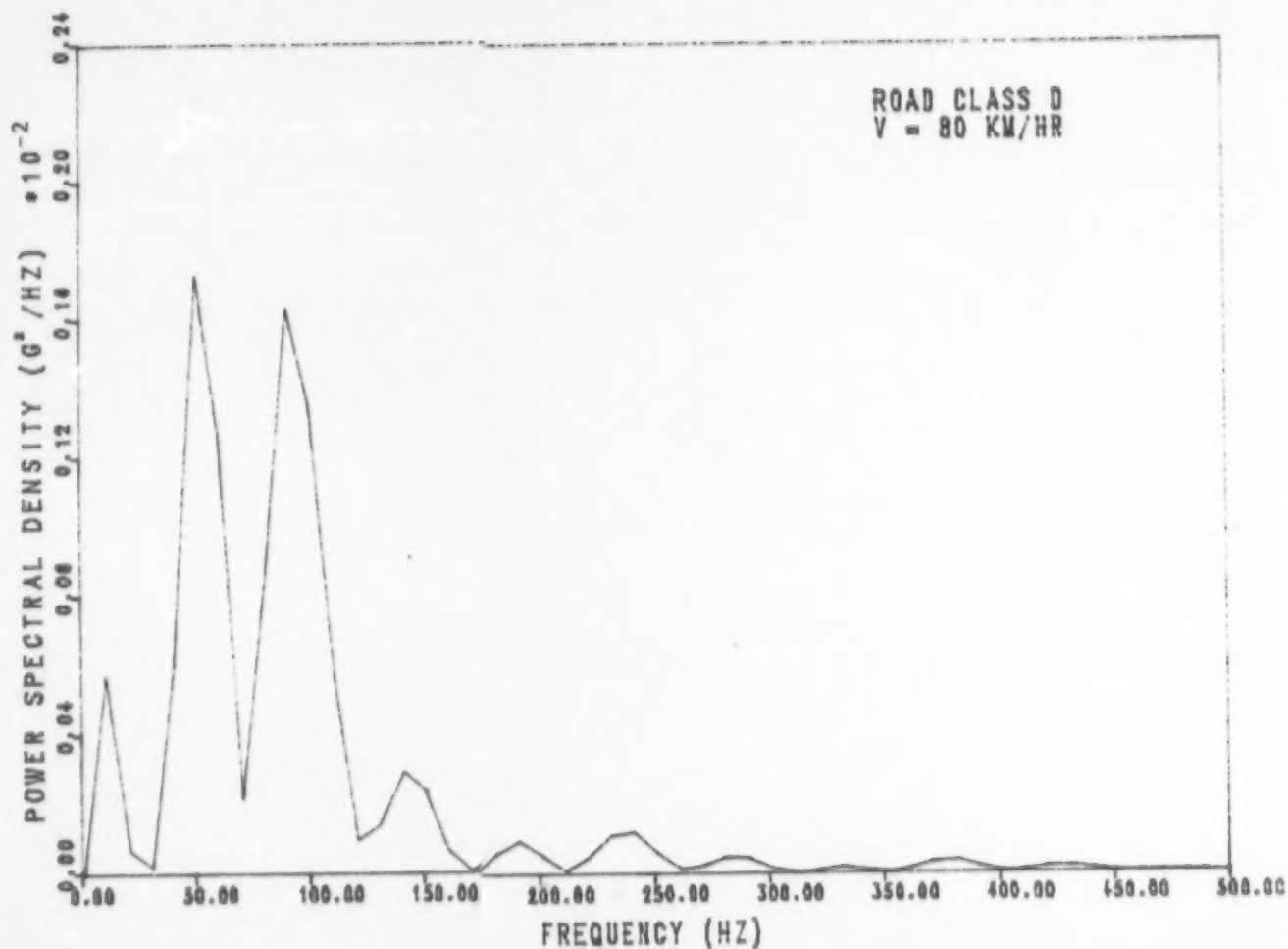


FIGURE 45  
THE POWER SPECTRAL DENSITY OF THE VERTICAL ACCELERATION  
OF THE MODULE STACK CENTRE OF GRAVITY VERSUS  
FREQUENCY FOR ROAD CLASS D

000079

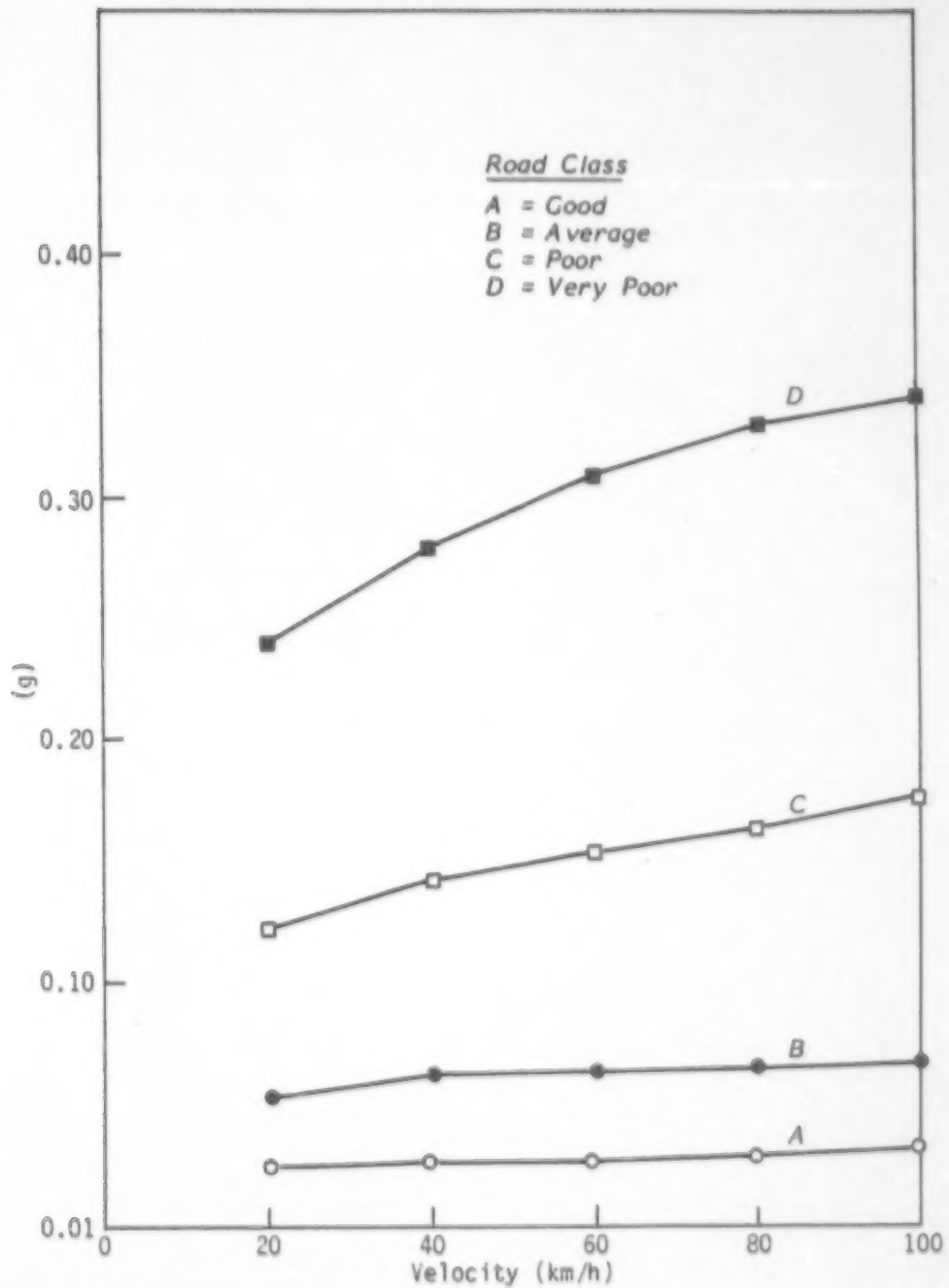


FIGURE 46  
R.M.S. OF VERTICAL ACCELERATION OF THE MODULE STACK C.G.  
VERSUS FORWARD VELOCITY

000080

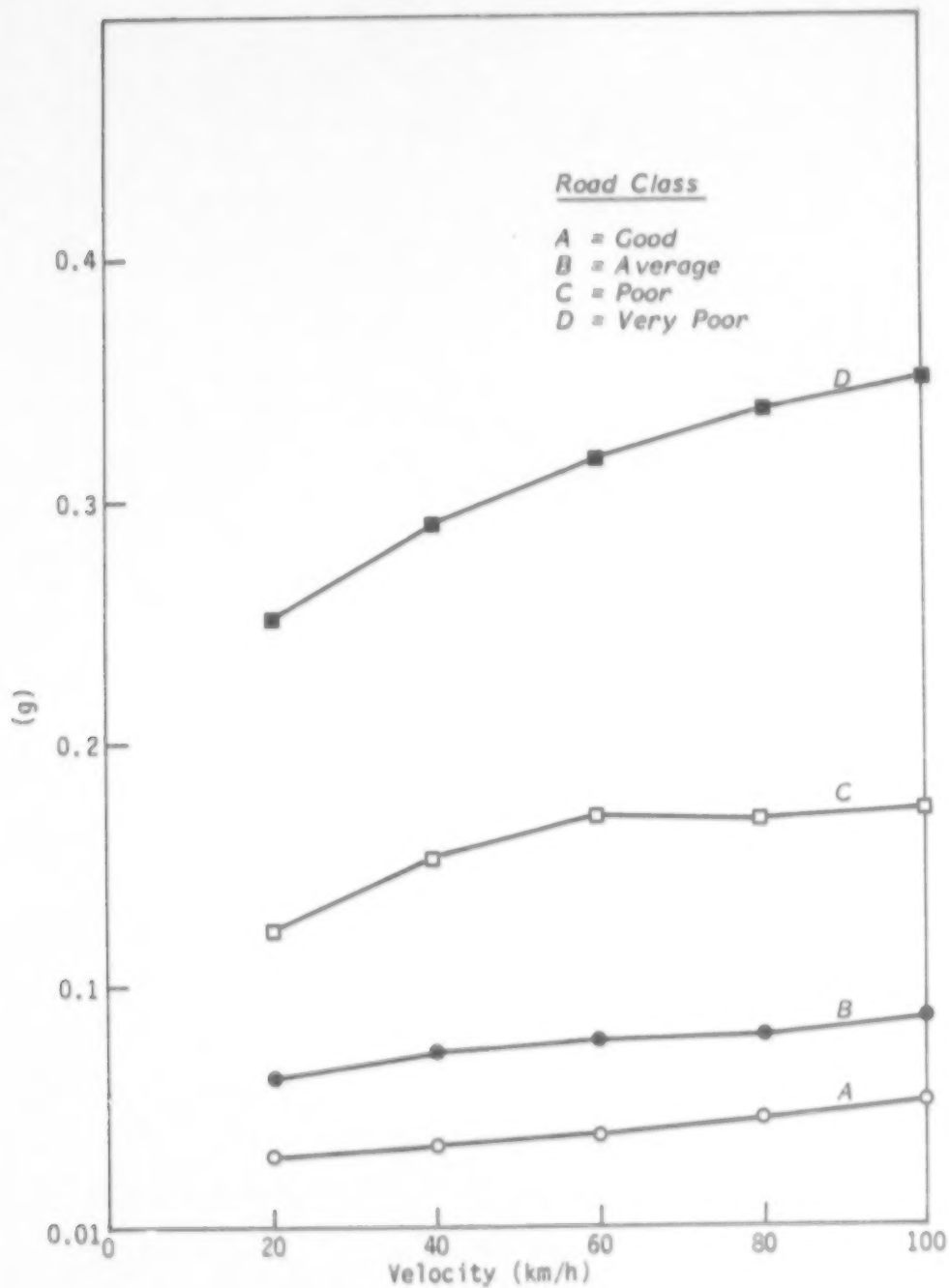


FIGURE 47  
R.M.S. OF VERTICAL ACCELERATION AT TIE DOWN  
CONNECTION WITH THE SEMI TRAILER

000001

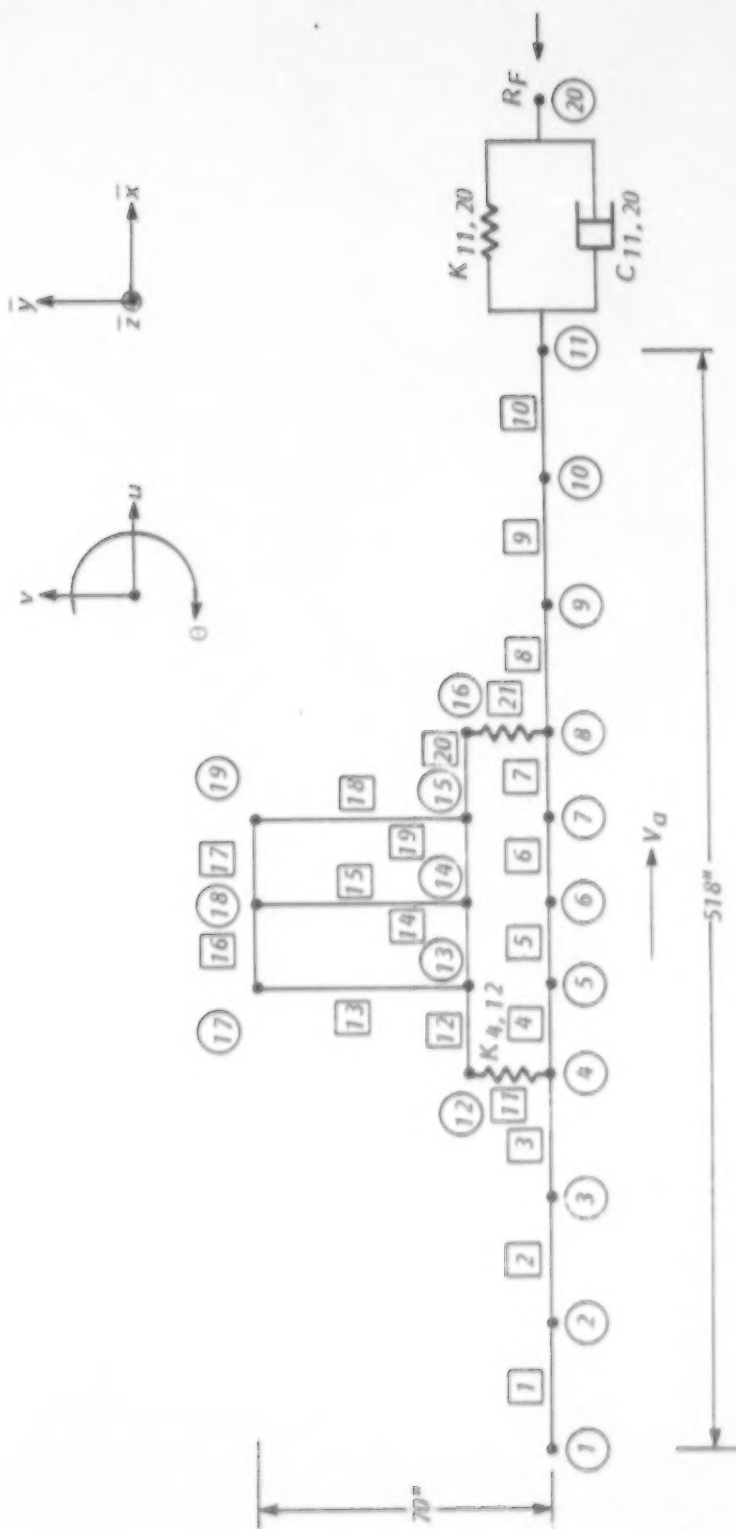


FIGURE 48  
STRUCTURAL MODELING TECHNIQUE FOR RAIL CAR, CASK, TRANSPORT  
FRAME, SHIPPING MODULES, AND COUPLER

000082

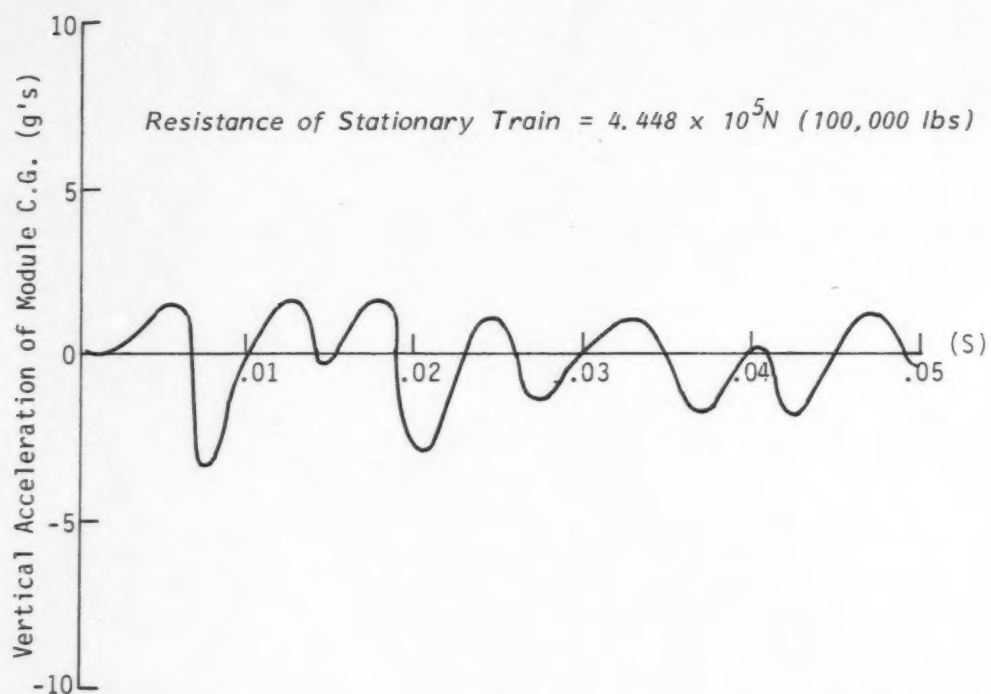


FIGURE 49  
ACCELERATION TIME PROFILE FOR RAIL CAR CARRYING 38-TONNE  
CARGO STRIKING 226.8-TONNE STATIONARY TRAIN AT 16 km/h

000083

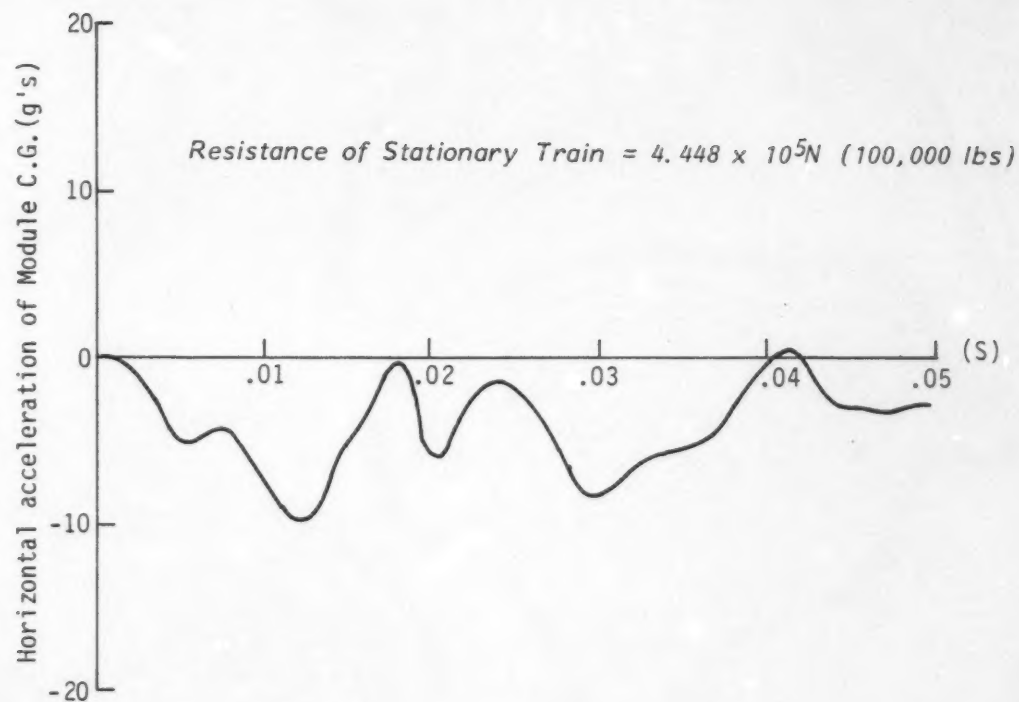


FIGURE 50  
ACCELERATION TIME PROFILE FOR RAIL CAR CARRYING  
38-TONNE CARGO STRIKING 226.8-TONNE STATIONARY  
TRAIN AT 16 km/h

000084

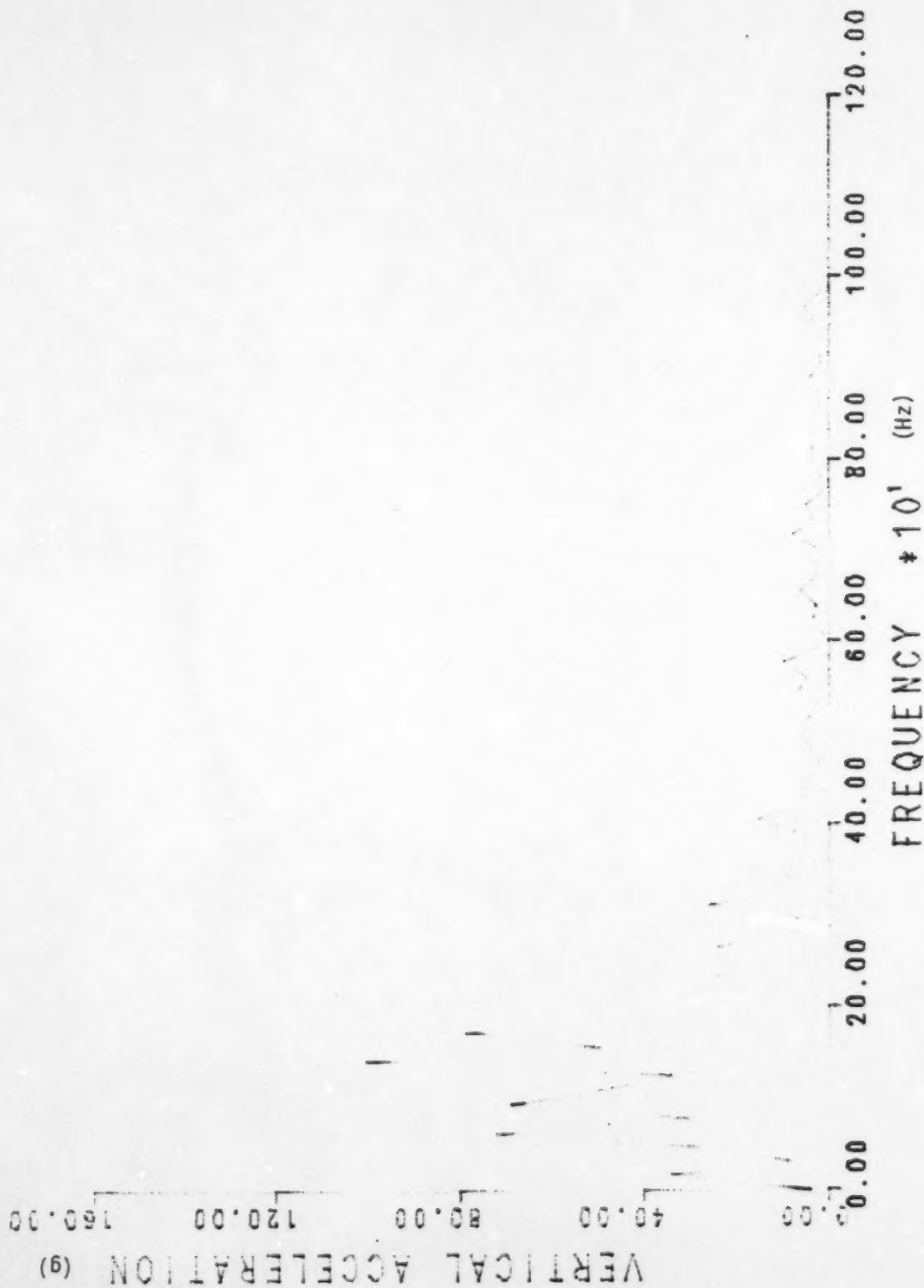


FIGURE 51  
VERTICAL ACCELERATION FREQUENCY PROFILE

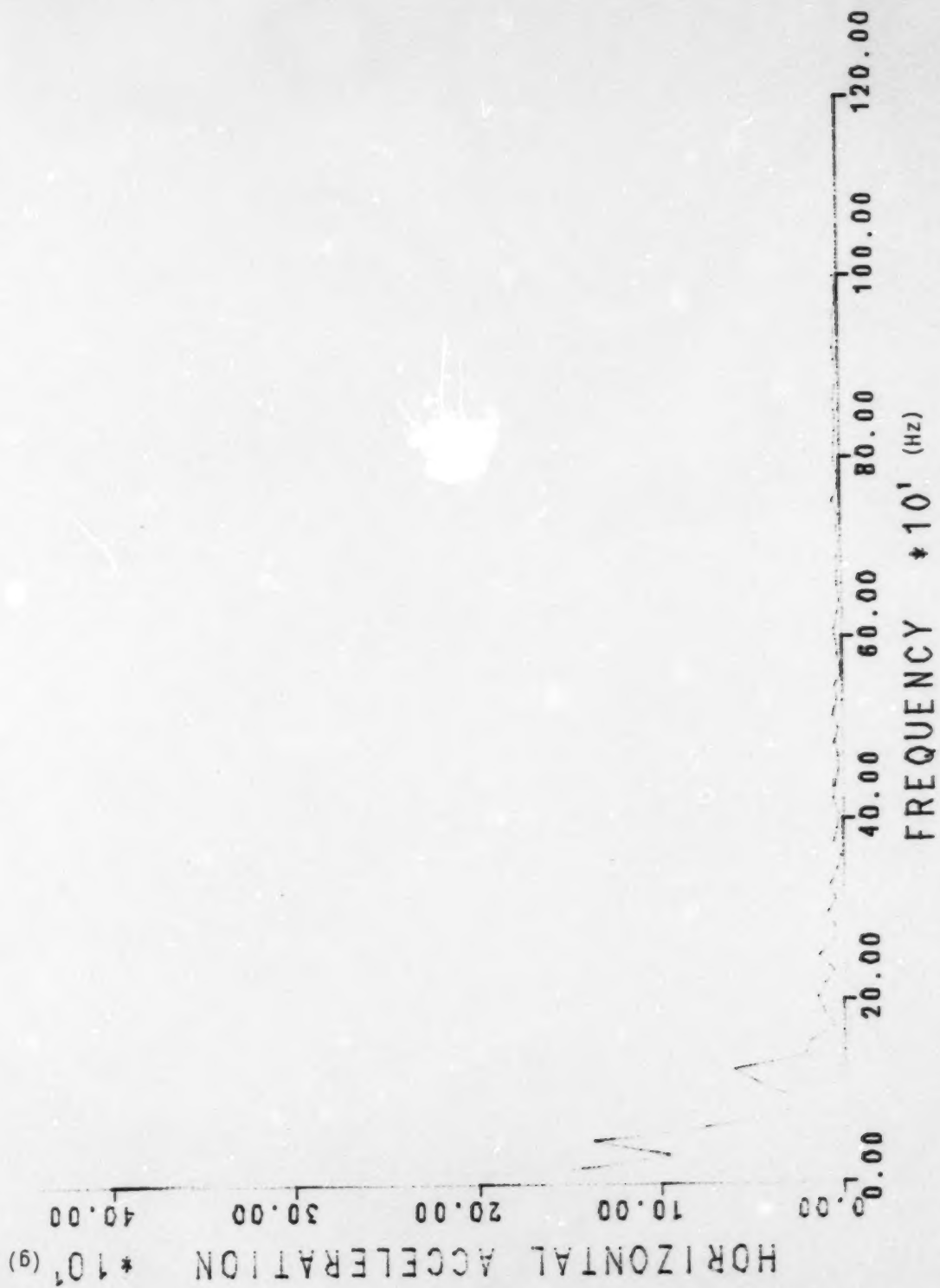


FIGURE 52  
HORIZONTAL ACCELERATION FREQUENCY PROFILE

000086

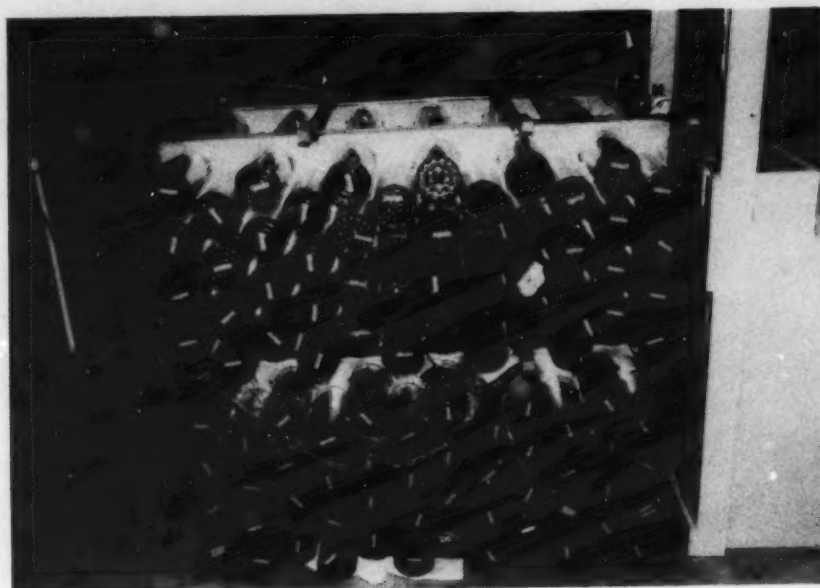


FIGURE 53  
FRONT VIEW OF TWO MODULE STACK



FIGURE 54  
SIDE VIEW OF TWO MODULE STACK

000087

162422 RD

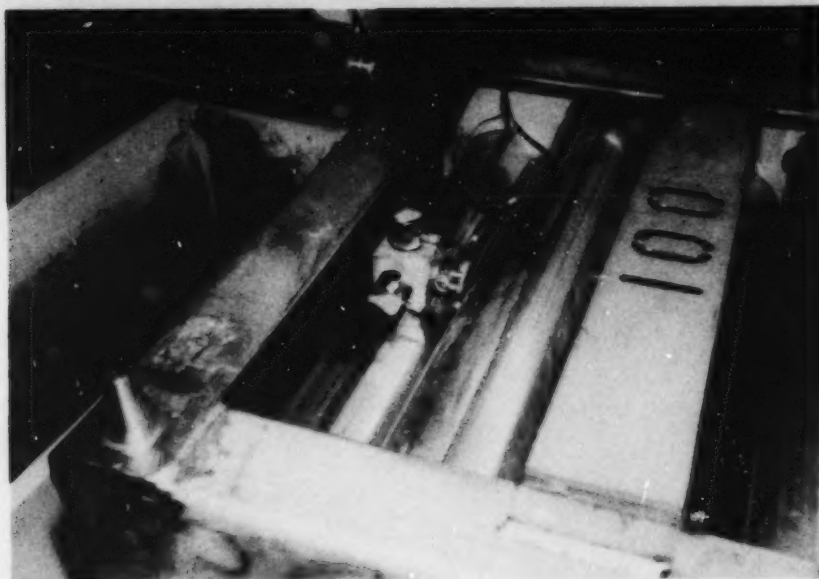


FIGURE 55  
ACCELERATION RESPONSE MEASUREMENT



FIGURE 56  
ACCELERATION RESPONSE MEASUREMENT

000088

162424 RD



FIGURE 57  
ACCELERATION RESPONSE MEASUREMENT

000089

162426 RD



FIGURE 58  
HAMMER USED FOR IMPACT TESTING

000090

162427 RD

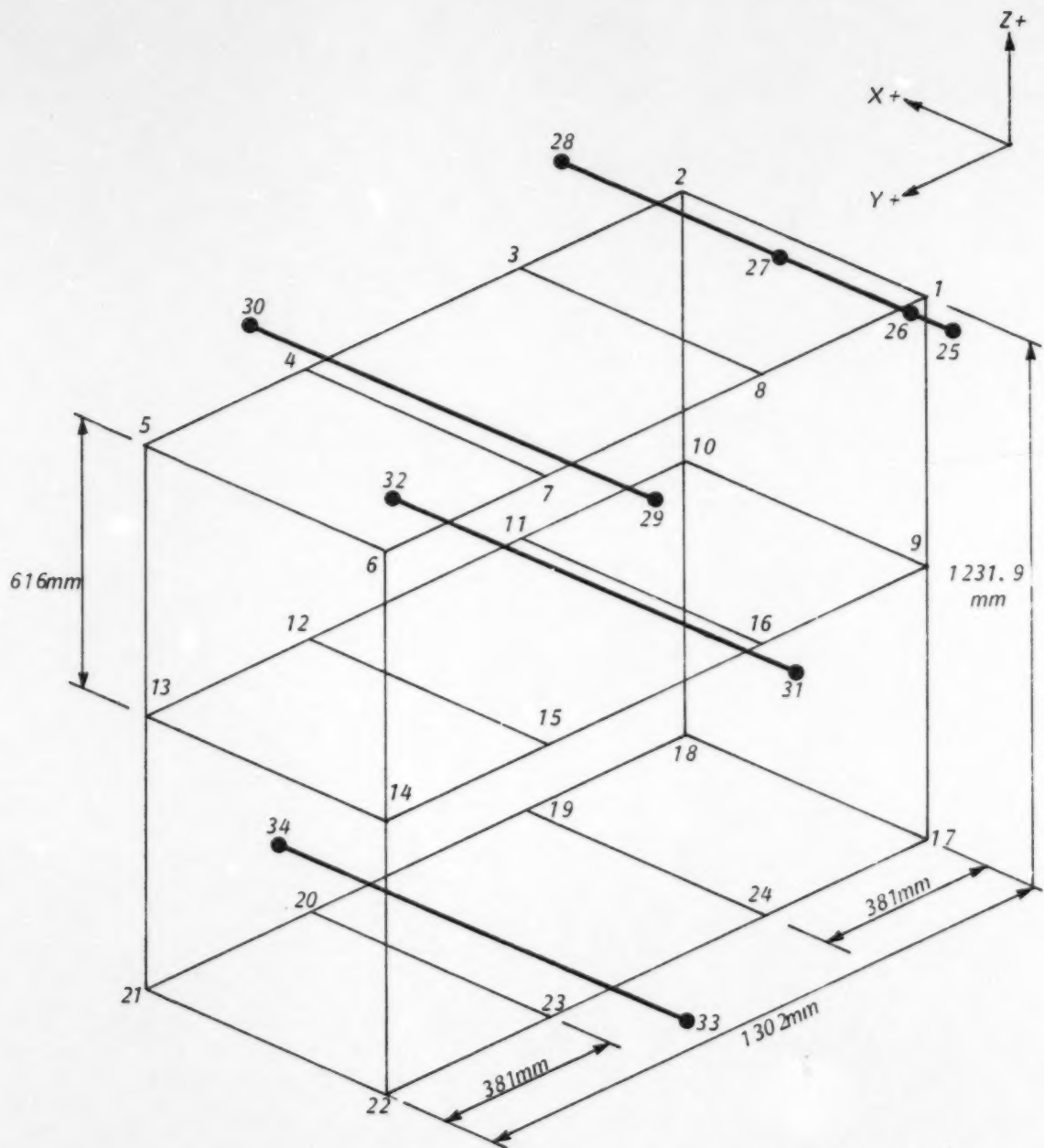


FIGURE 59  
MODULE STACK MODEL

000091

162428 RD

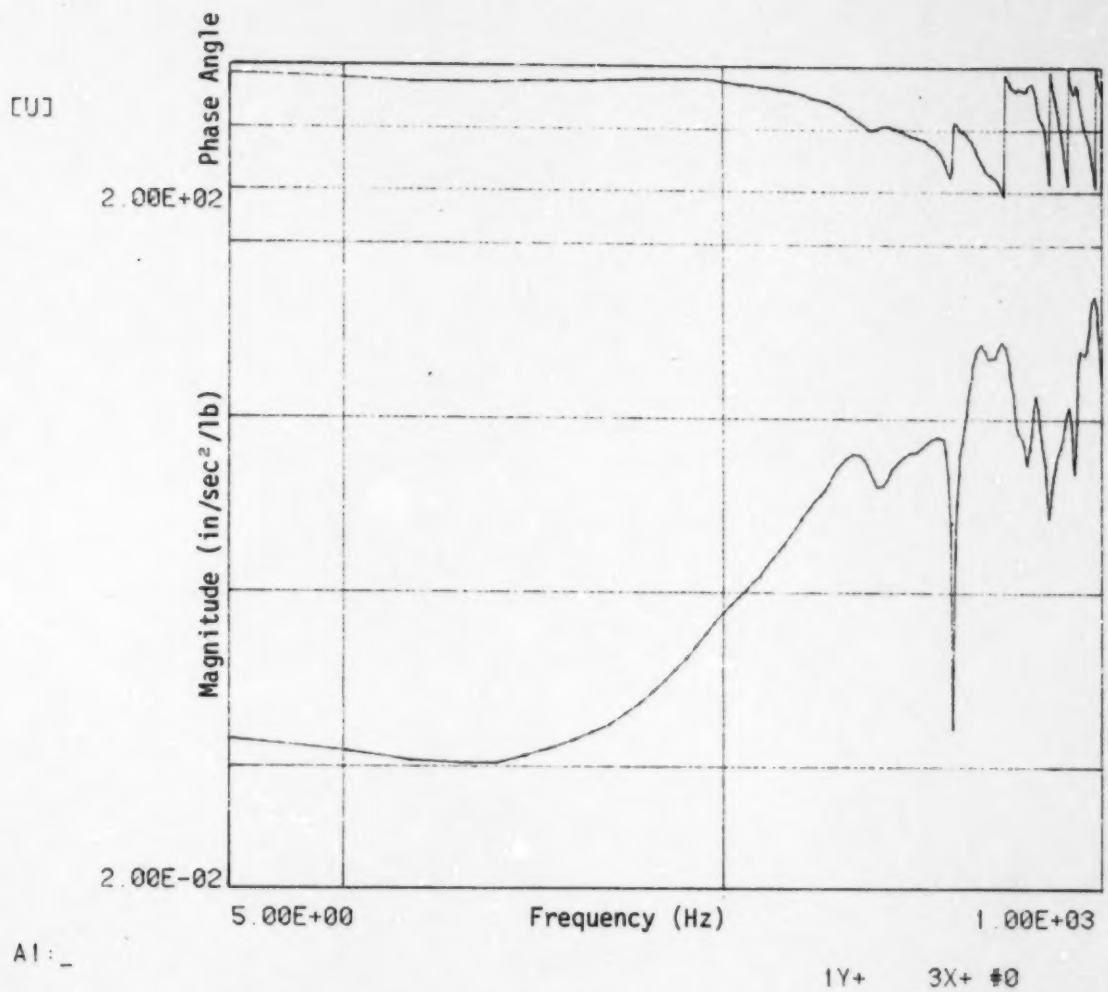


FIGURE 60  
MEASURED TRANSFER FUNCTION

000092

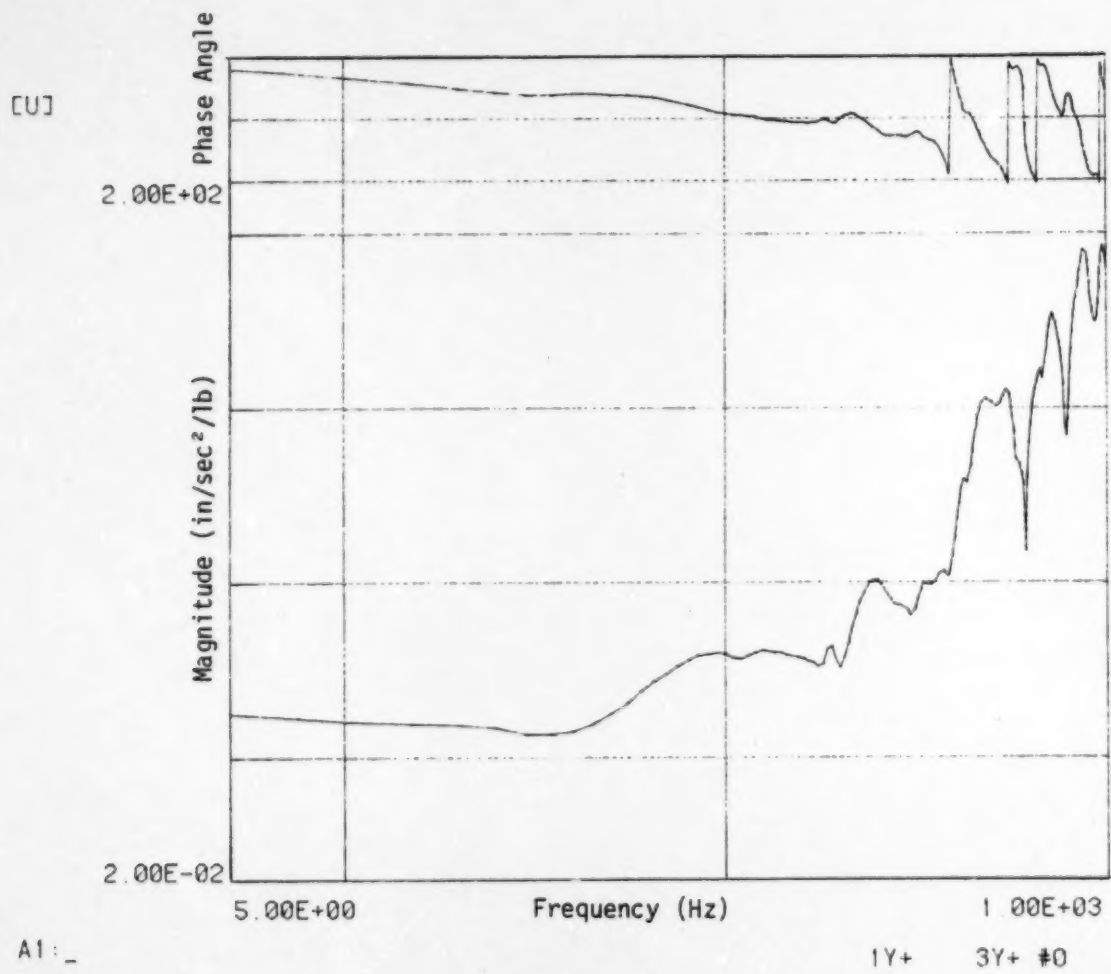


FIGURE 61  
MEASURED TRANSFER FUNCTION

000093

162430 RD

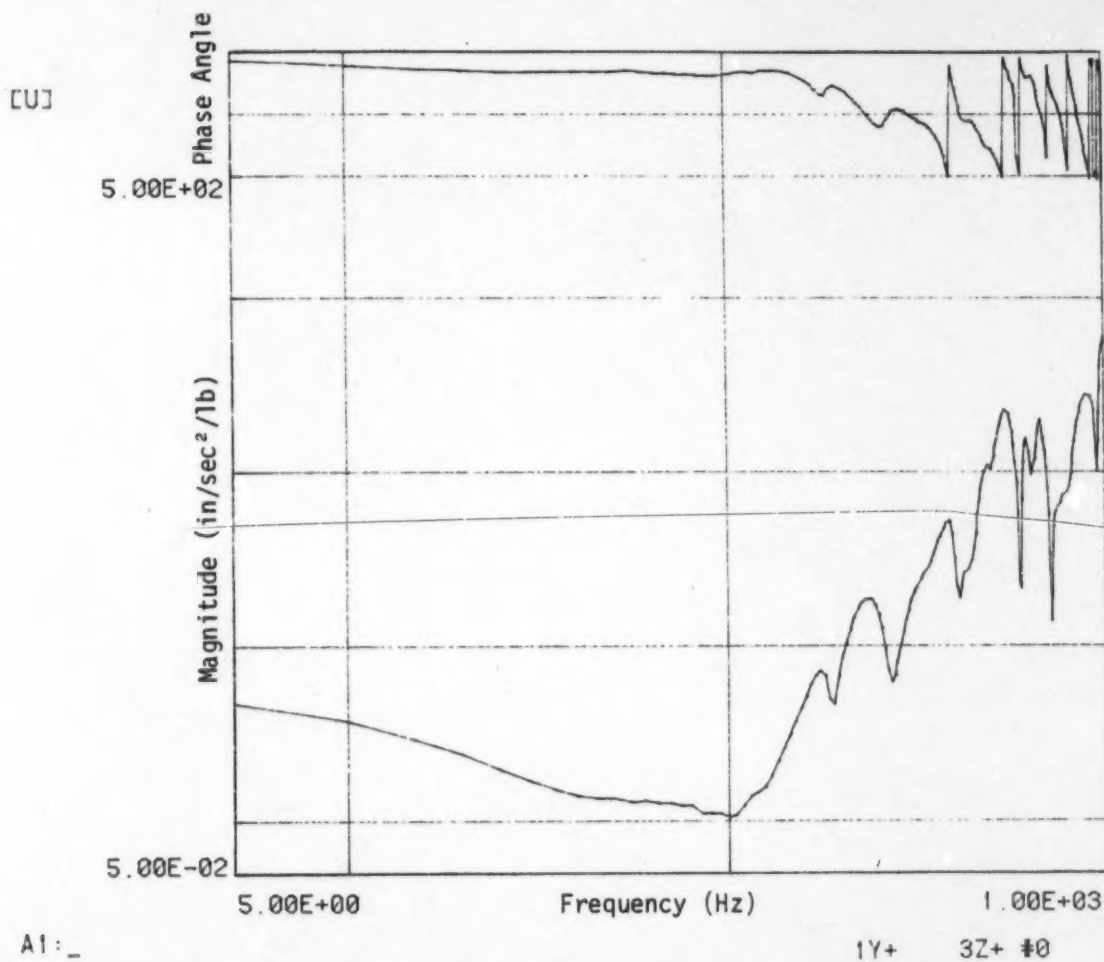
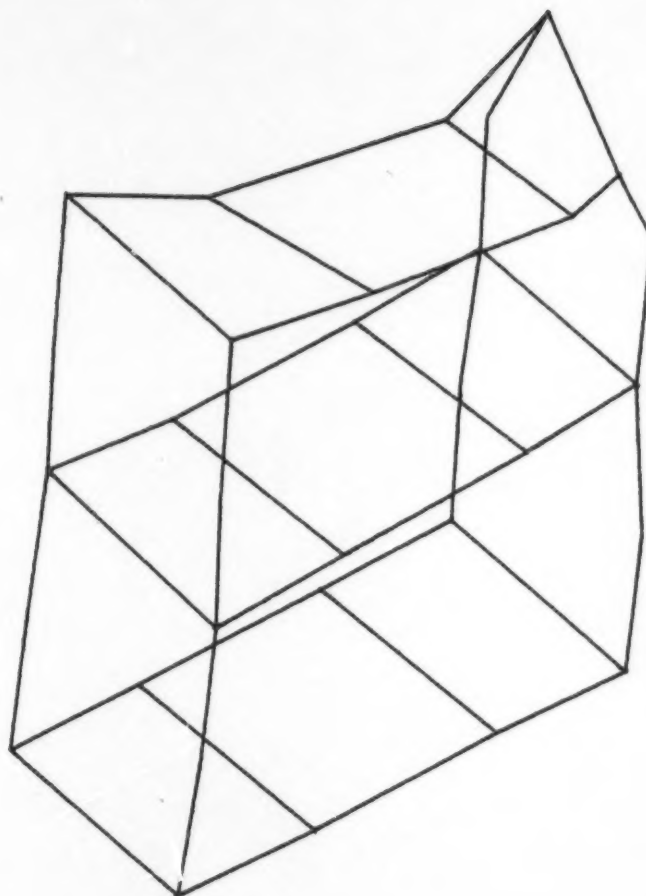


FIGURE 62  
MEASURED TRANSFER FUNCTION

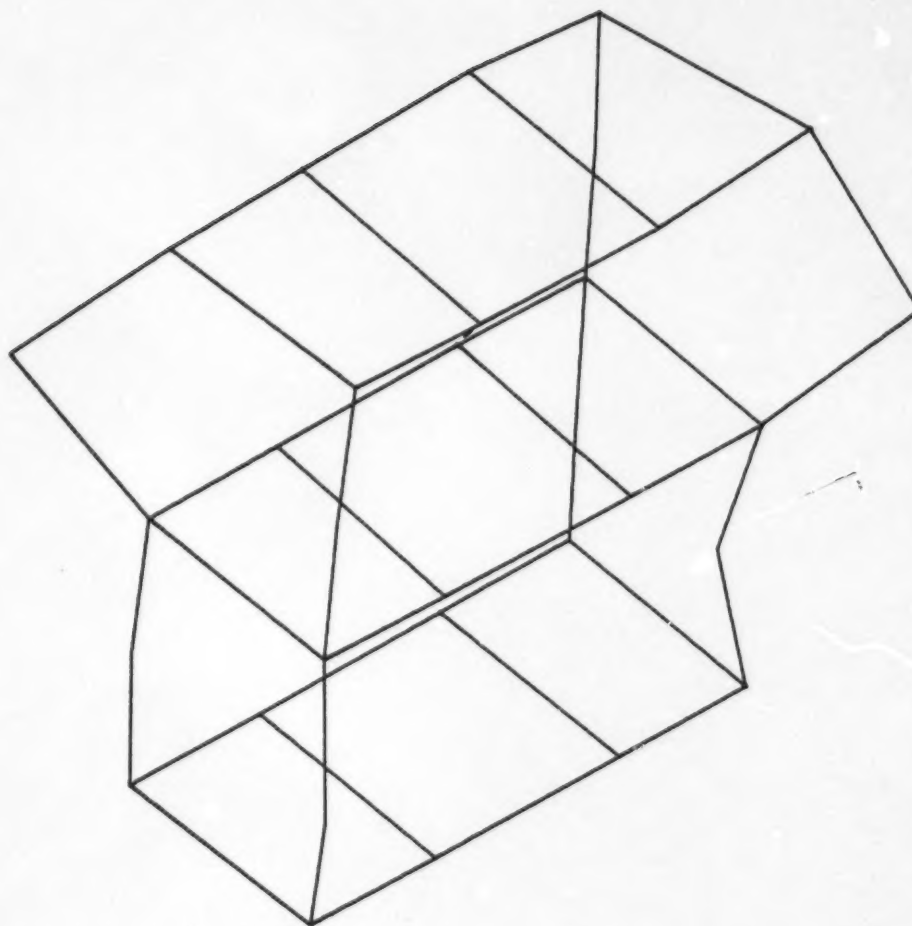
00009'



1: 1Y+ COMP.F= 196.791 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 63  
MODE SHAPE NUMBER 1

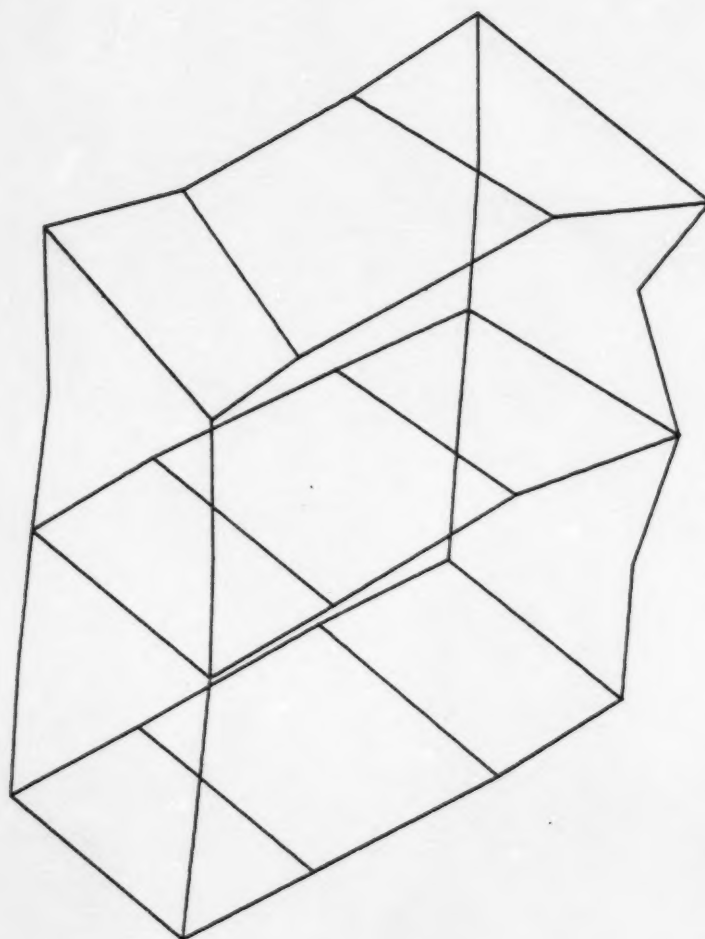
000095



2: 1Y+ COMP.F= 253.803 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 64  
MODE SHAPE NUMBER 2

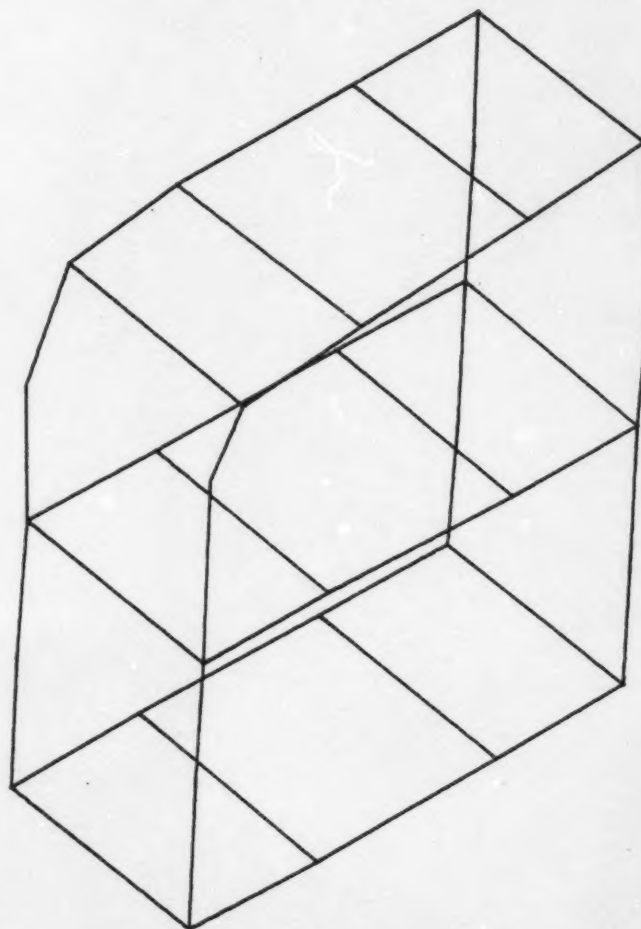
000096



3: 1Y+ COMP.F= 393.350 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 65  
MODE SHAPE NUMBER 3

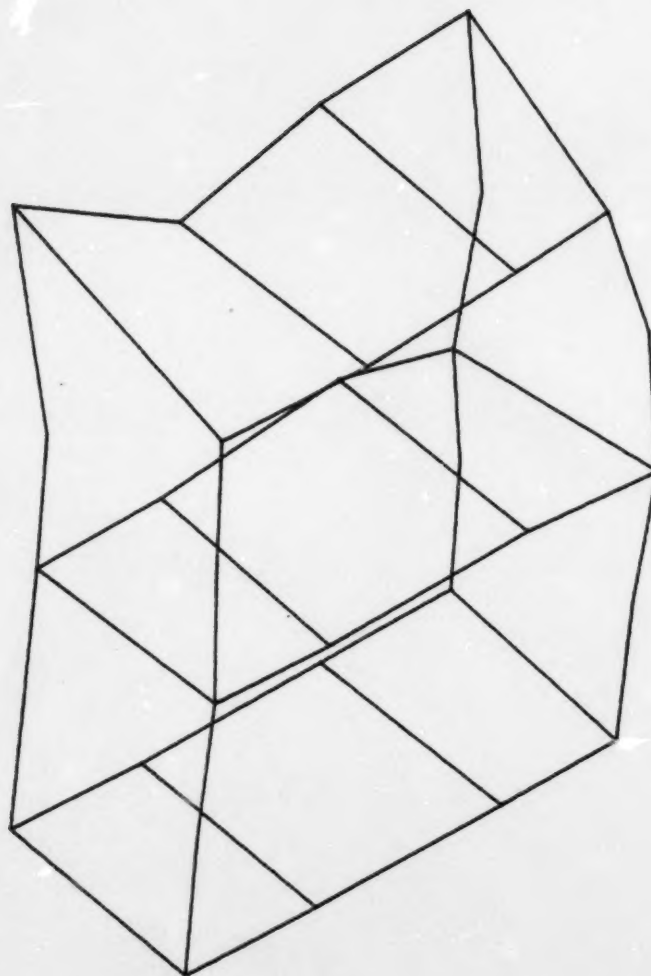
000097



4: 1Y+ COMP.F= 315.429 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 66  
MODE SHAPE NUMBER 4

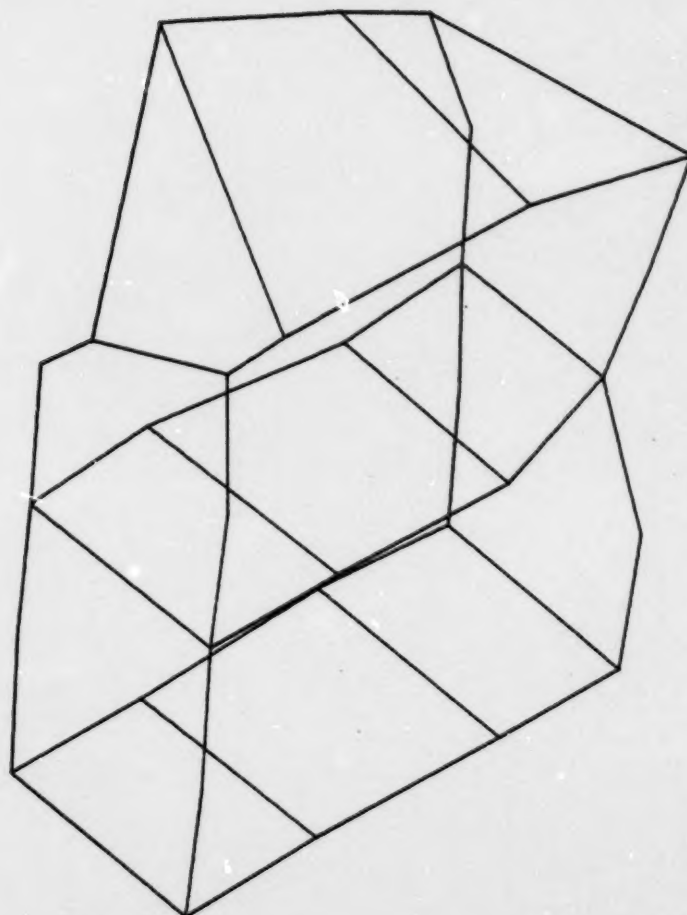
000098



5: 1Y+ COMP.F= 486.923 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 67  
MODE SHAPE NUMBER 5

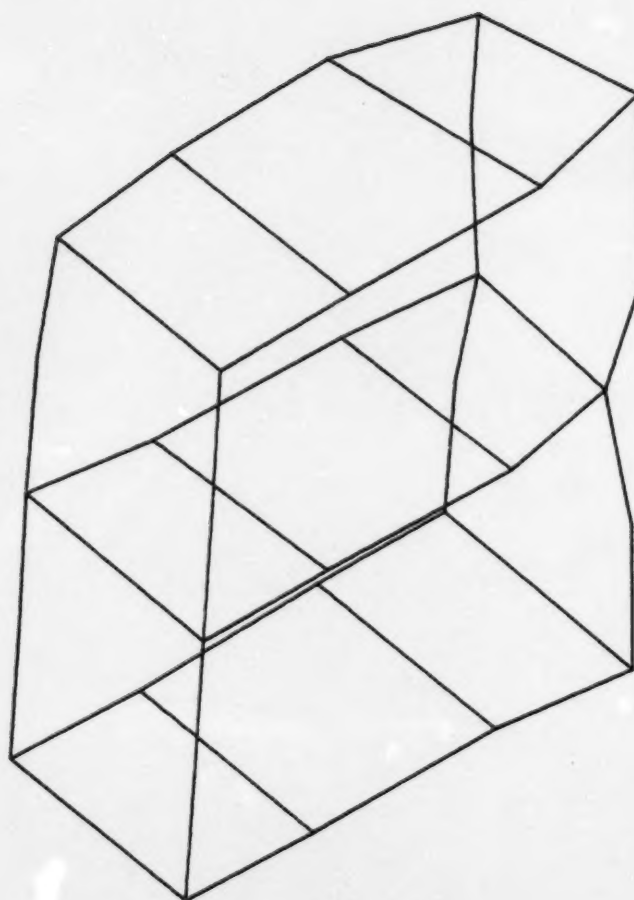
000099



6: 1Y+ COMP.F= 518.046 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 68  
MODE SHAPE NUMBER 6

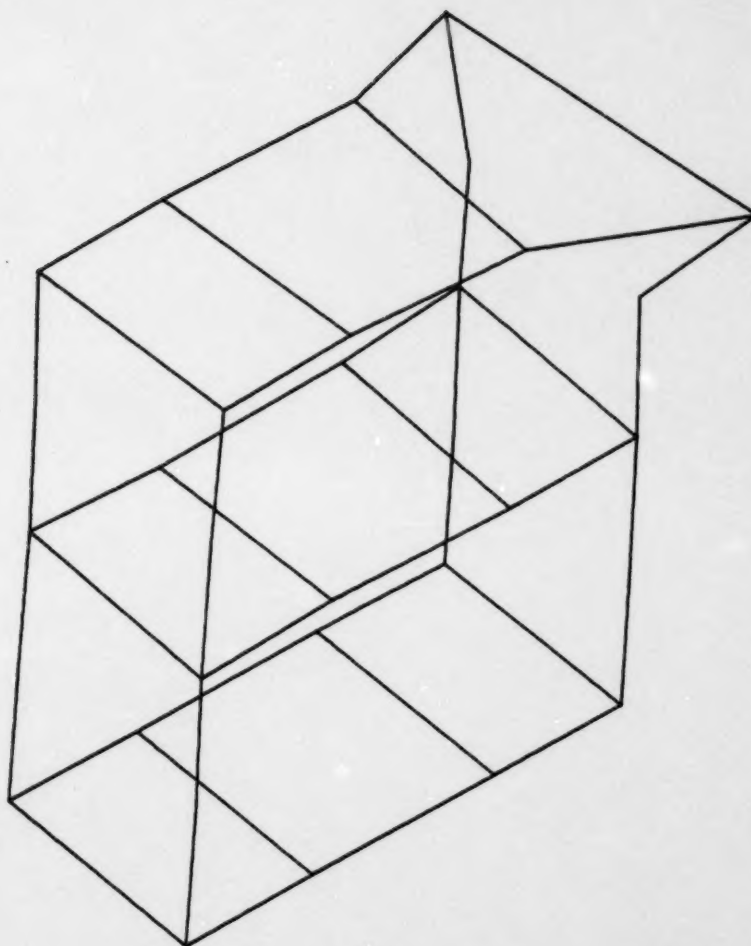
000100



7: 1Y+ COMP.F= 646.335 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 69  
MODE SHAPE NUMBER 7

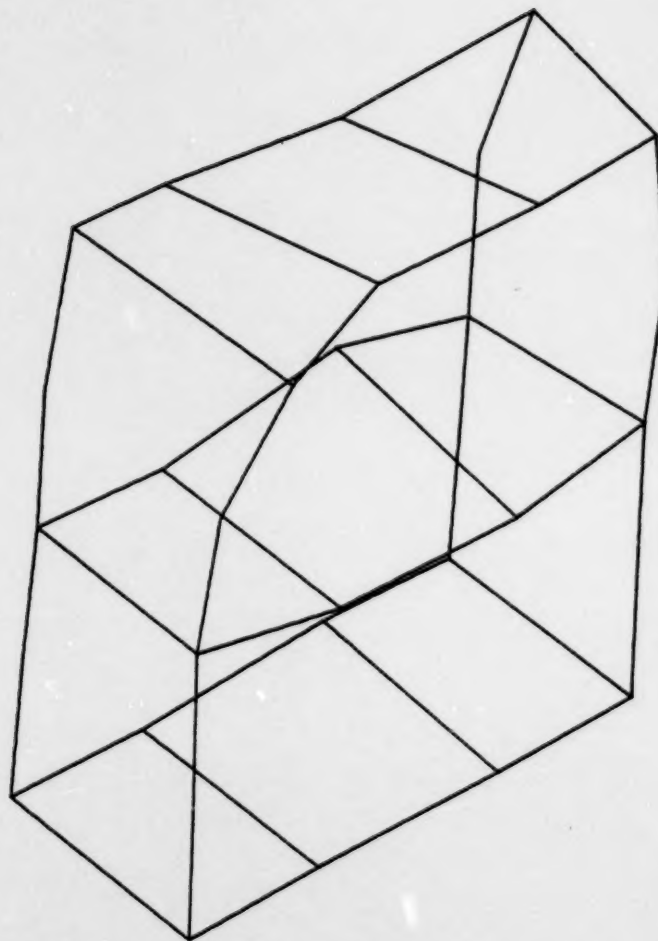
000101



8: 1Y+ COMP.F= 742.175 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 70  
MODE SHAPE NUMBER 8

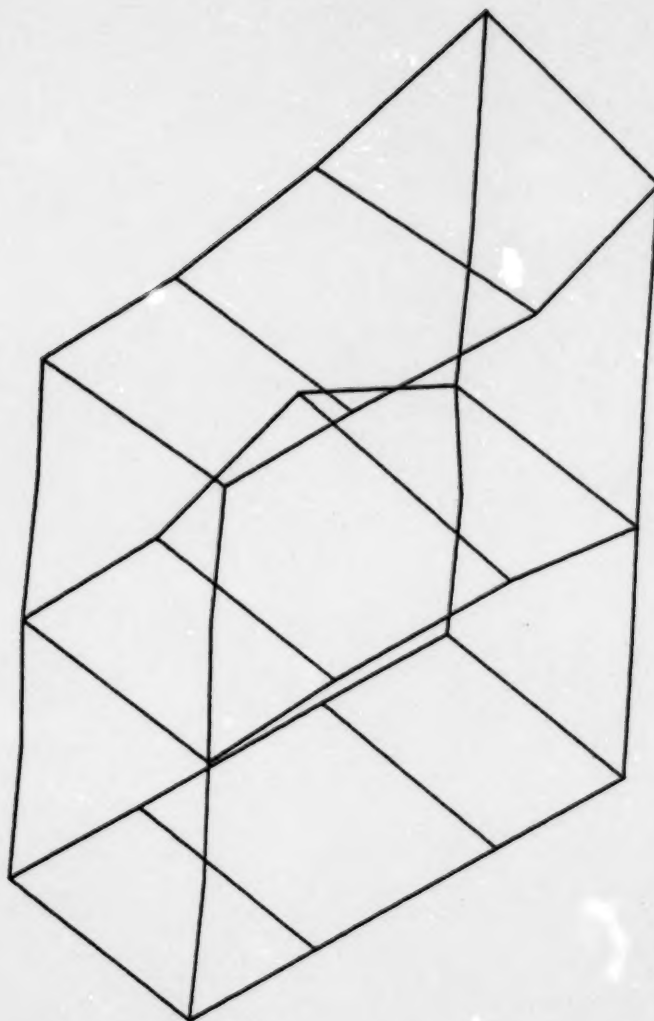
000102



9: 1Y+ COMP.F= 859.272 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 71  
MODE SHAPE NUMBER 9

000103



10: 1Y+ COMP.F= 946.819 HZ ( -1.0. 1.0. 1.0. 115.0)=VIEW

FIGURE 72  
MODE SHAPE NUMBER 10

000104

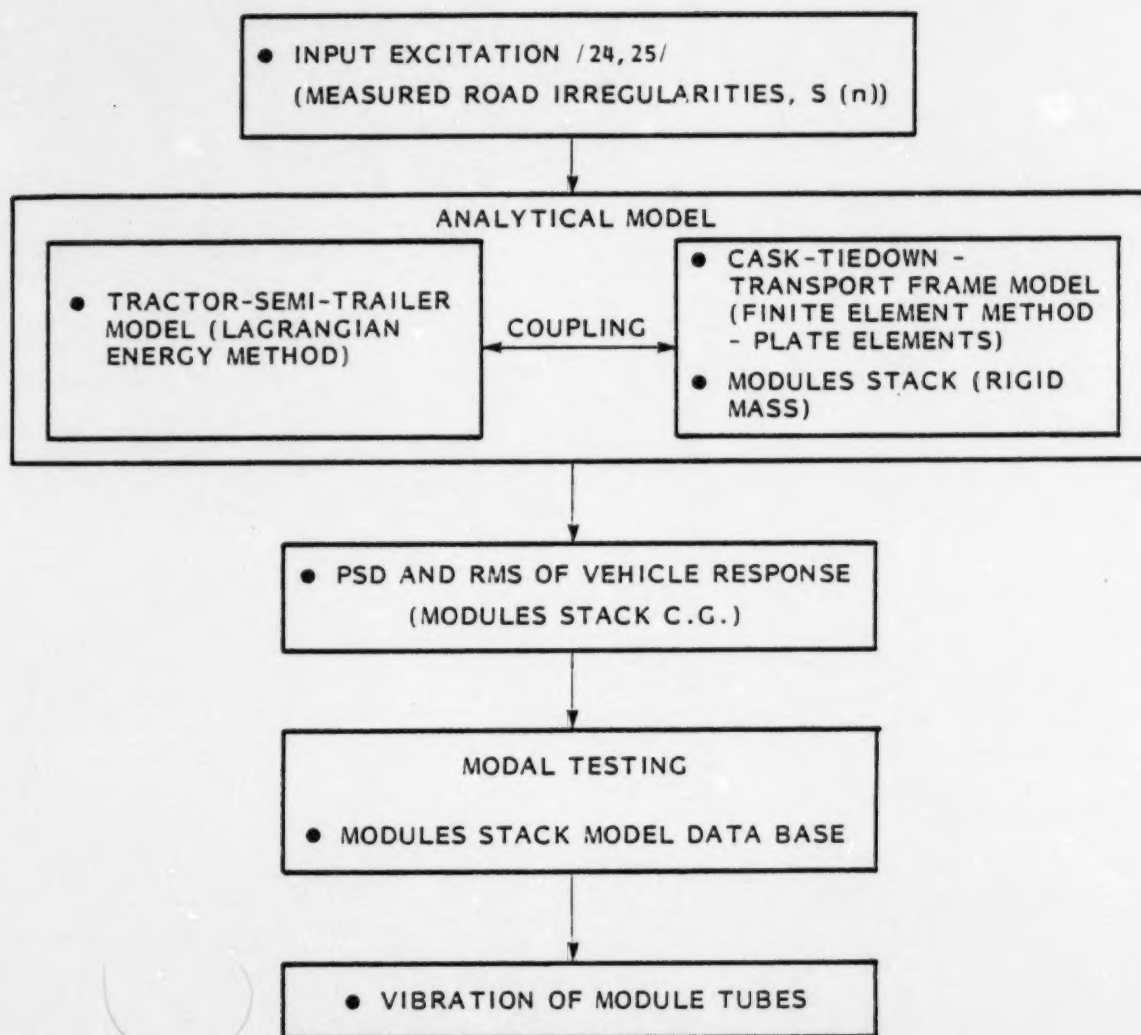


FIGURE 73  
MODULE ANALYSIS: ROAD TRANSPORTATION MODEL

000105

5.00E-03

7

9

000106

5.00E-07

1.00E-01 Frequency (Hz)

FREQUENCY (Hz)

1.00E-01

5.00E-07

4.44E+02

AMPLIFIED RESPONSE

000072-000000

FREQRESP-000E

000072-000000

1X+

57+ #1

FIGURE 7#

VERTICAL VIBRATION RESPONSE OF MODULE TUBE

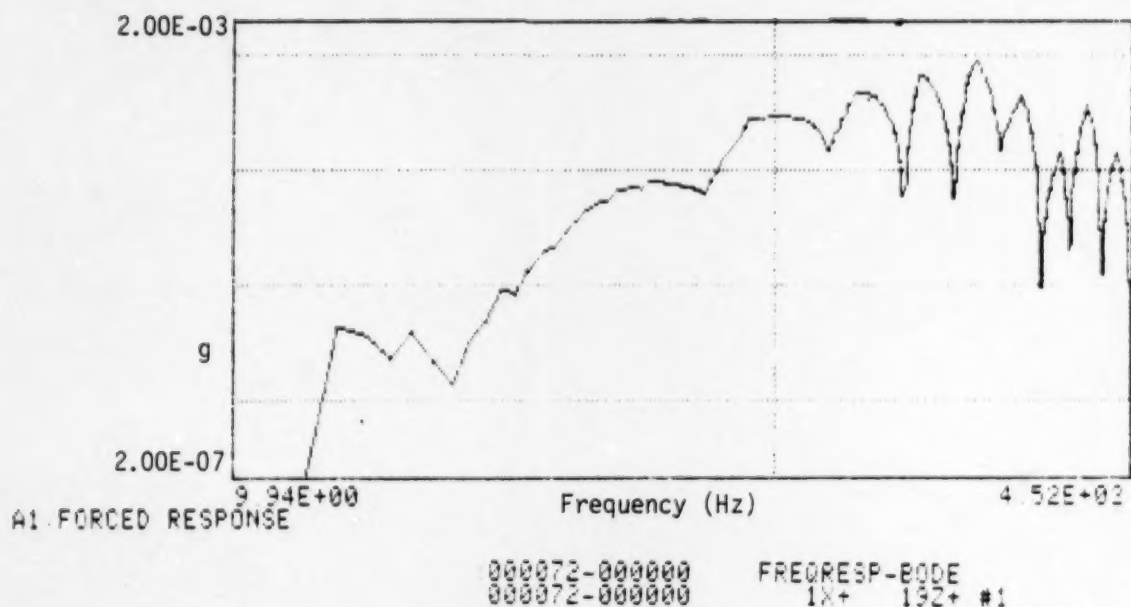


FIGURE 75  
VERTICAL VIBRATION RESPONSE OF MODULE TUBE

000107

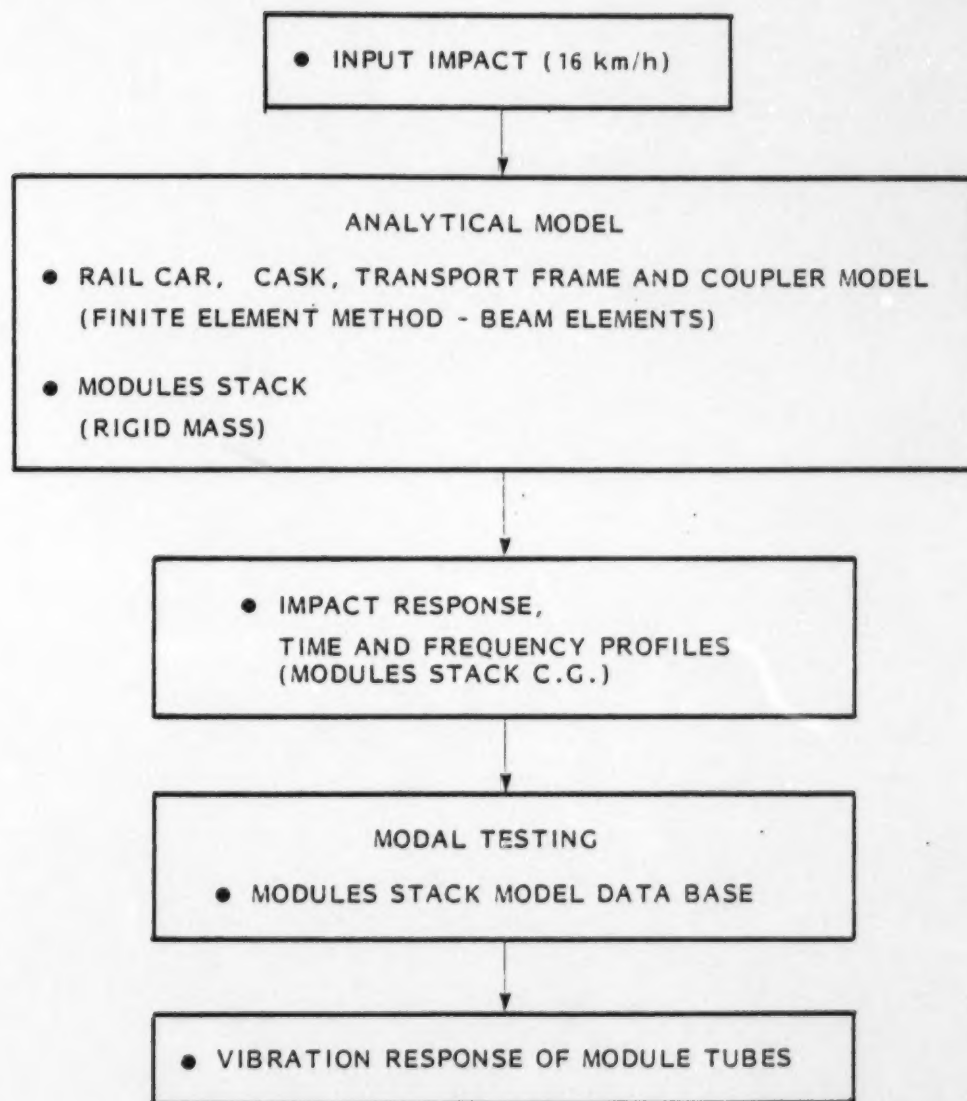


FIGURE 76  
MODULE ANALYSIS: RAILCAR END IMPACT MODEL

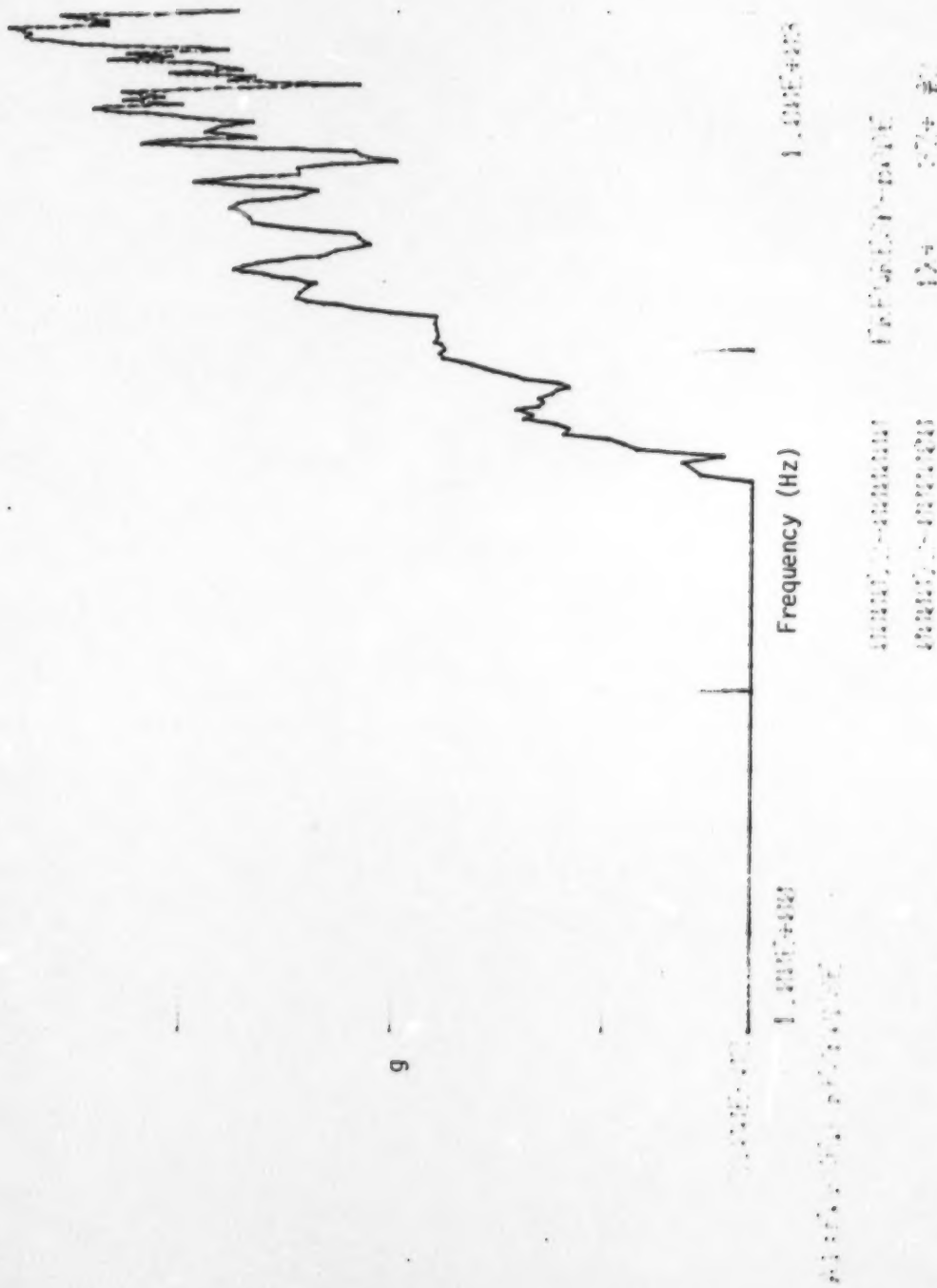


FIGURE 77  
VERTICAL SHOCK RESPONSE OF MODULE TUBE



FIGURE 78  
VERTICAL SHOCK RESPONSE OF MODULE TUBE

000110

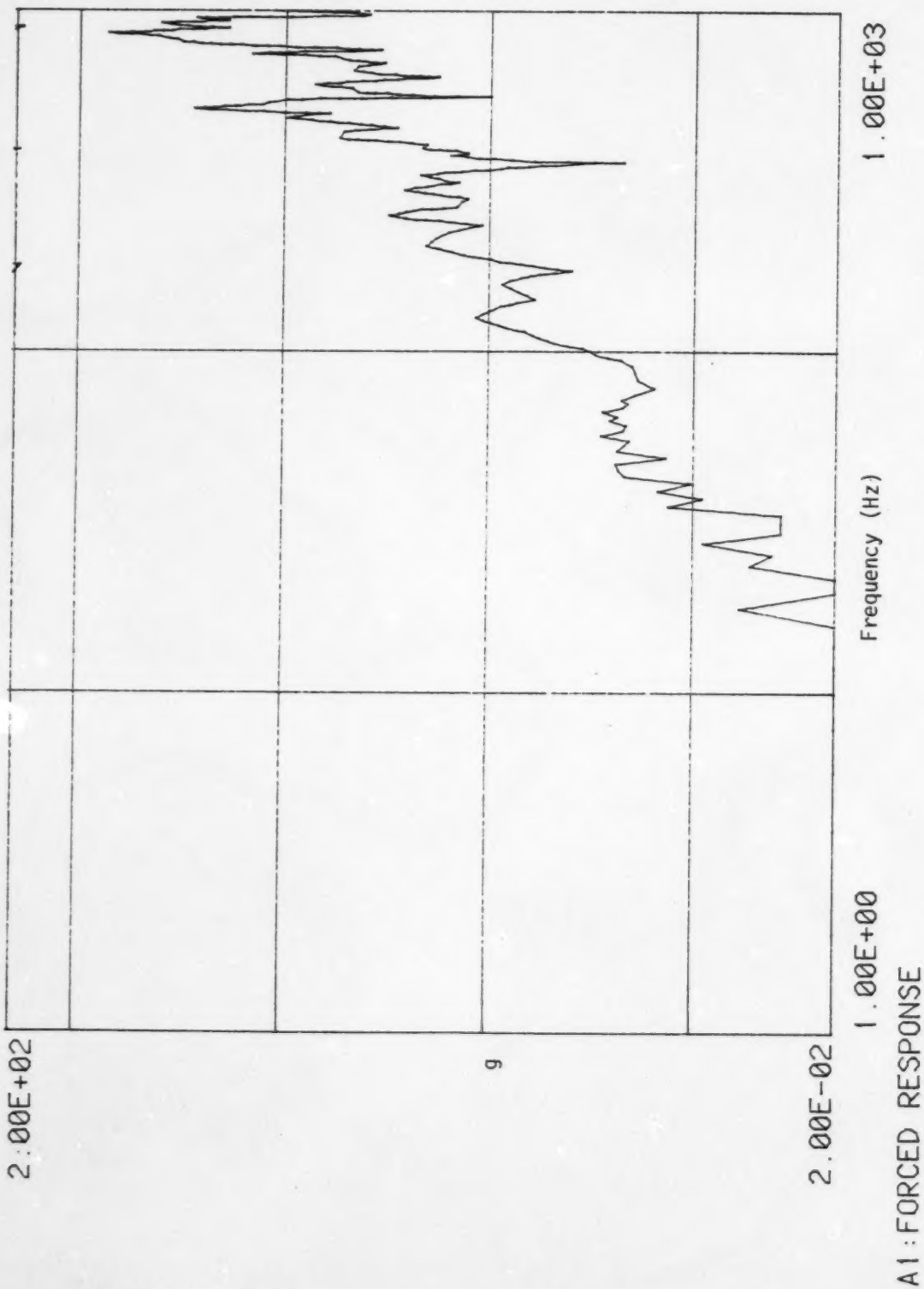
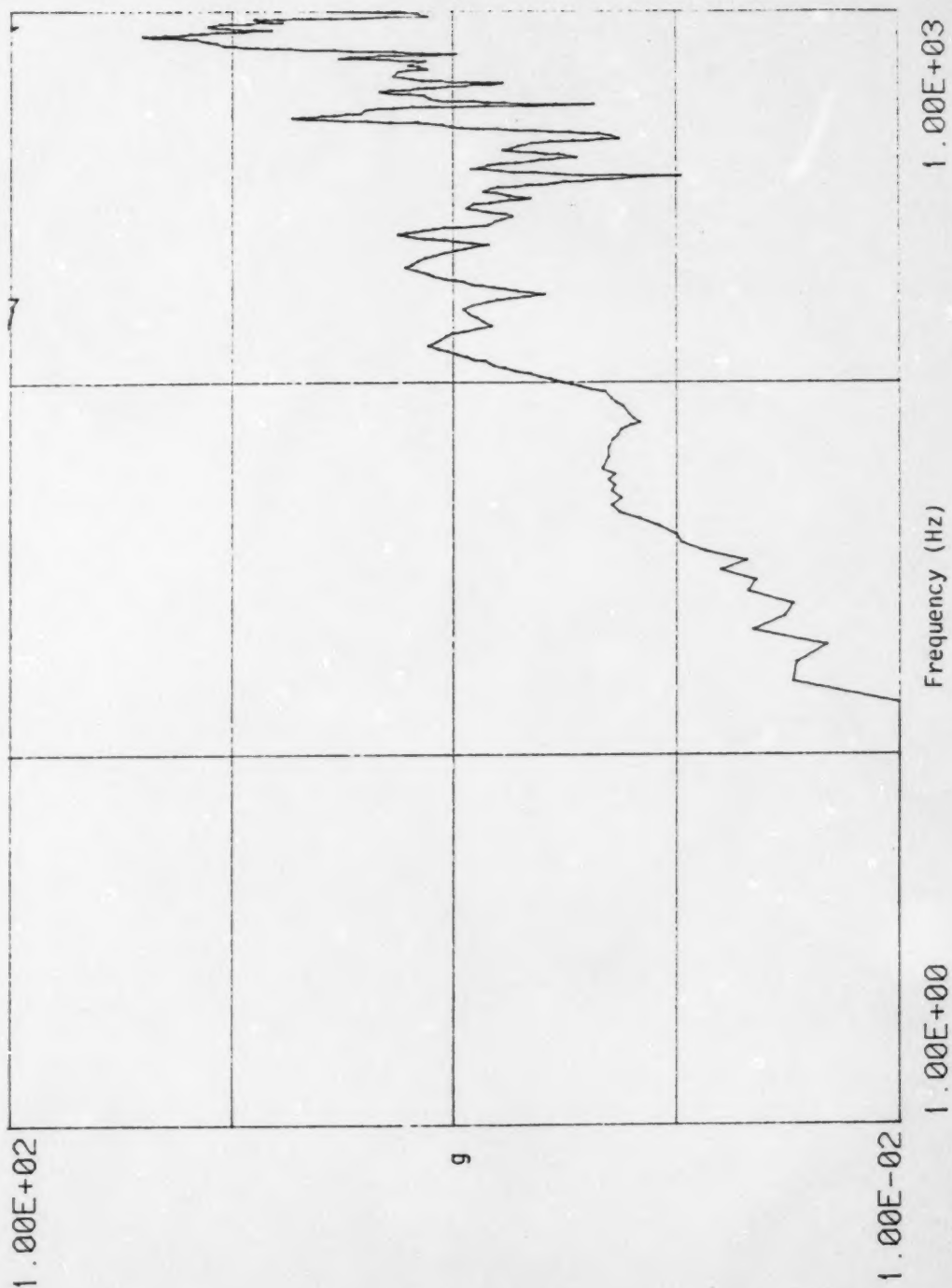


FIGURE 79

HORIZONTAL SHOCK RESPONSE OF MODULE TUBE

000111



A1:FORCED RESPONSE

000072-000000 FREQRESP-BODE  
 000072-000000 1X+ 19Z+ #3

FIGURE 80  
 HORIZONTAL SHOCK RESPONSE OF MODULE TUBE

000112

## APPENDIX I

### EQUATIONS OF MOTION OF THE ARTICULATED VEHICLE MODEL

The equations of motion for the articulated vehicle model shown in Figure 1 and subject to the road forces are given below. Due to the articulation between the tractor and semitrailer units, there are two equations of kinematic constraints,

$$Y_s = Y_t + b_5 \theta_t + b_4 \theta_s. \quad (I.1)$$

and

$$X_s = X_t + a_1 \theta_t - a_2 \theta_s. \quad (I.2)$$

One dynamical equation revealed an ignorable coordinate which provided the integrated equation,

$$X_t = - \frac{M_5}{M_4 + M_5} (a_1 \theta_t - a_2 \theta_s) \quad (I.3)$$

Therefore, there are six independent coordinates. They are  $Y_t$ ,  $\theta_t$ ,  $\theta_s$ ,  $Y_1$ ,  $Y_2$  and  $Y_3$

Tractor Body Vertical

$$\begin{aligned} & (M_t + M_s) \ddot{Y}_t + M_s b_5 \ddot{\theta}_t + M_s b_4 \ddot{\theta}_s + (c_1 + c_2 + c_3) \dot{Y}_t \\ & + (-b_1 c_1 + b_2 c_2 + b_5 c_3) \dot{\theta}_t + (b_3 + b_4) \dot{\theta}_s - c_1 \dot{Y}_1 \\ & - c_2 \dot{Y}_2 - c_3 \dot{Y}_3 + (k_1 + k_2 + k_3) Y_t \\ & + (-b_1 k_1 + b_2 k_2 + b_5 k_3) \theta_t \\ & - k_1 Y_1 - k_2 Y_2 - k_3 Y_3 \\ & = 0 \end{aligned} \quad (I.4)$$

### Tractor Body Pitch

$$\begin{aligned}
 & (I_t + b_5^2 M_s + \frac{M_4 M_5}{M_4 + M_5} a_1^2) \ddot{\theta}_t + b_5 M_s \ddot{y}_t + (b_4 b_5 M_s - \frac{M_4 M_5}{M_4 + M_5} a_1 a_2) \ddot{\theta}_s \\
 & + (-b_1 c_1 + b_2 c_2 + b_5 c_3) \dot{y}_t + (b_1^2 c_1 + b_2^2 c_2 + b_5^2 c_3) \dot{\theta}_t \\
 & + (b_3 + b_4) b_5 c_3 \dot{\theta}_s + b_1 c_1 \dot{y}_1 - b_2 c_2 \dot{y}_2 - b_5 c_3 \dot{y}_3 \\
 & + (-b_1 k_1 + b_2 k_2 + b_5 k_3) y_t + (b_1^2 k_1 + b_2^2 k_2 + b_5^2 k_3) \theta_t \\
 & + (b_3 + b_4) b_5 k_3 \theta_s + b_1 k_1 y_1 - b_2 k_2 y_2 - b_5 k_3 y_3 \\
 & = 0
 \end{aligned} \tag{I.5}$$

### Semitrailer Body Pitch

$$\begin{aligned}
 & (I_s + b_4^2 M_s + \frac{M_4 M_5}{M_4 + M_5} a_1^2) \ddot{\theta}_s + b_4 M_s \ddot{y}_t + (b_4 b_5 M_s - \frac{M_4 M_5}{M_4 + M_5} a_1 a_2) \ddot{\theta}_t \\
 & + (b_3 + b_4) c_3 \dot{y}_t + b_5 (b_3 + b_4) c_3 \dot{\theta}_t + (b_3 + b_4)^2 c_3 \dot{\theta}_s \\
 & - (b_3 + b_4) c_3 \dot{y}_3 + (b_3 + b_4) k_3 y_t + b_5 (b_3 + b_4) k_3 \theta_t \\
 & + (b_3 + b_4)^2 k_3 \theta_s - (b_3 + b_4) k_3 y_3 \\
 & = 0
 \end{aligned} \tag{I.6}$$

### Tractor Front Unsprung Mass Vertical

$$\begin{aligned}
 & M_1 \ddot{y}_1 - c_1 \dot{y}_t + b_1 c_1 \dot{\theta}_t + c_1 \dot{y}_1 - k_1 y_t + b_1 k_1 \theta_t \\
 & + k_1 y_1 + c_{t1} (\dot{y}_1 - \dot{w}_1) + k_{t1} (y_1 - w_1) = 0
 \end{aligned} \tag{I.7}$$

### Tractor Rear Unsprung Mass Vertical

$$\begin{aligned}
 & M_2 \ddot{y}_2 - c_2 \dot{y}_t + b_2 c_2 \dot{\theta}_t + c_2 \dot{y}_2 - k_2 y_t + b_2 k_2 \theta_t \\
 & + k_2 y_2 + c_{t2} (\dot{y}_2 - \dot{w}_2) + k_{t2} (y_2 - w_2) = 0
 \end{aligned} \tag{I.8}$$

Semitrailer Unsprung Mass Vertical

$$\begin{aligned}
 M_3 \ddot{Y}_3 - c_3 \dot{Y}_t - b_5 c_3 \dot{\theta}_t - (b_3 + b_4) c_3 \dot{\theta}_s + c_3 \dot{Y}_3 \\
 - k_3 Y_t - b_5 k_3 \theta_t - (b_3 + b_4) k_3 \theta_s + k_3 Y_3 \\
 + c_{t3} (\dot{Y}_3 - \dot{w}_3) + k_{t3} (Y_3 - w_3) = 0
 \end{aligned}
 \tag{I.9}$$

where

$$M_4 = M_t + M_1 + M_2$$

$$M_5 = M_s + M_3$$

## APPENDIX II

### MATHEMATICAL DERIVATION OF INPUT SPECTRAL DENSITY MATRIX

Suppose that the road vehicle train moves with a constant forward velocity,  $v$ , along a road profile given by  $\omega(x)$ , which can be considered to be a member function of a stationary Gaussian random process,  $\{\omega(x)\}$ , and the argument of which is the instantaneous location on the tract,  $x$ . Let the mean value  $E[\omega(x)] = 0$ .

So, as the vehicle is moving along the rough road, it is subjected to  $m$  vertically imposed spatial displacement functions,  $\omega_j(x)$ , ( $j = 1, \dots, m$ ), one at each wheel set. Let the imposed spatial displacement inputs to the mathematical model,  $\omega_j(x)$ , have a correlation function matrix,  $R_{\omega}(\delta) = E[\omega^*(x_1)\omega'(x_2)]$ , where  $\delta \equiv x_2 - x_1$ , and a spatial spectral density function matrix  $S_{\omega}(n)$ .

Conversion of the spatial inputs,  $\omega_j(x)$ , to temporal inputs,  $w_j(t)$ , is accomplished through a simple transformation whereby the spatial coordinate  $x$  is replaced by  $vt$ . Therefore, the relation between  $\omega_j(x)$  and  $w_j(t)$  is given by

$$\omega_j(vt) = w_j(t), \quad j = 1, \dots, m \quad (\text{II.1})$$

Suppose that the corresponding input temporal correlation function matrix and temporal spectral density matrix are given by  $R_w(\tau) = E[\bar{w}^*(t_2)\bar{w}'(t_1)]$  and  $S_w(f)$ , respectively.

The relationship between the spatial and temporal correlation functions and their corresponding spectral density functions can be derived as follows.

By considering that  $f = nv$  and  $\delta = v\tau$ , the correlation function,  $R_w(\tau)$ , is given by

$$\begin{aligned} R_w(\tau) &= E[\bar{w}^*(t) \bar{w}'(t+\tau)] \\ &= E[\bar{\omega}^*(vt) \bar{\omega}'(vt+v\tau)] \\ &= E[\bar{\omega}^*(x) \bar{\omega}'(x+\delta)] \\ &= R_{\omega}(\delta) \end{aligned} \quad (\text{II.2})$$

The spectral density function,  $S_w(f)$ , is given by

$$\begin{aligned} S_w(f) &= \int_{-\infty}^{\infty} 2 R_w(\tau) \exp(-i2\pi f\tau) d\tau \\ &= \frac{1}{v} \int_{-\infty}^{\infty} 2 R_w(\delta) \exp(-i2\pi n\delta) d\delta \\ &= \frac{1}{v} S_w(n) \end{aligned} \quad (II.3)$$

In the matrix  $S_w(f)$ ,  $S_{jk}(f)$  is the input displacement spectral density to the vehicle at the tire contact point. When  $j = k$ ,  $S_{jk}(f)$  represents the direct power spectral density of  $w_j(t)$  and in this case  $S_{jj}(f) = 1/v S_{jj}(n)$ . When  $j \neq k$ ,  $S_{jk}(f)$  represents the cross-spectral density of  $w_j(t)$  and  $w_k(t)$ .

To compute the cross spectral density,  $S_{jk}(f)$ , assume that the vehicle rear wheels follow the same profile as the front wheel, but separated by a constant time delay,  $\tau_{jk} = l_{jk}/v$ , where  $l_{jk}$  is the spacing between  $j$ th and  $k$ th wheel. The cross-correlation function of any two wheels,  $j$  and  $k$ , is given by

$$\begin{aligned} R_{jk}(\tau) &= E[w_j^*(t)w(t+\tau)] \\ &= E[w_j^*(t)w_j(t+\tau-l_{jk}/v)] \\ &= R_{jj}(\tau - l_{jk}/v) \end{aligned} \quad (II.4)$$

and hence the cross-spectrum

$$\begin{aligned} S_{jk}(f) &= \int_{-\infty}^{\infty} 2 R_{jk}(\tau - l_{jk}/v) \exp(-i2\pi f\tau) d\tau \\ &= \int_{-\infty}^{\infty} 2 R_{jk}(\tau - l_{jk}/v) \exp(-i2\pi f(\tau - l_{jk}/v)) d(\tau - l_{jk}/v) \\ &\quad \exp(-i2\pi f l_{jk}/v) \\ &= S_{jj}(f) \exp(-i2\pi f l_{jk}/v) \end{aligned} \quad (II.5)$$

Combining Equation (II.3) and (II.5) and considering the property of cross-spectra:

$$S_{jk}(f) = S_{kj}^*(f), \quad (II.6)$$

the temporal input displacement spectral density,  $S_w(f)$ , may be written in terms of the surface profile as follows:

$$S_w(f) = \frac{s(n)}{v} \begin{bmatrix} 1 & e^{-i\beta_{12}} & \dots & e^{-i\beta_{1m}} \\ e^{-i\beta_{12}} & 1 & & e^{-i\beta_{2m}} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & 1 \\ e^{-i\beta_{1m}} & e^{-i\beta_{2m}} & \dots & 1 \end{bmatrix} \quad (II.7)$$

where  $\beta_{jk} = 2\pi f \lambda_{jk}/v$

To compute the excitation power spectral density function matrix,  $S_f(f)$ , the damping and elastic properties of the tires are used. In this case the excitation correlation function is given by  $R_f(\tau) = E[k_f^* \bar{w}^*(t) \bar{w}'(t+\tau) K_f' + C_f^* \bar{w}^*(t) \bar{w}'(t+\tau) C_f']$  and by taking the Fourier transformation of  $R_f(\tau)$ , the excitation spectral density matrix,  $S_f(f)$ , is given by

$$S_f(f) = (K_f + i2\pi f C_f)^* S_w(f) (K_f + i2\pi f C_f)' \quad (II.8)$$

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